

7.4.2. Judging the scope of quantifier phrases

In the examples we have just been looking at, we were free to choose the order in which we analyzed quantifier phrases; but that is not always possible. A change in the order of analysis will change the relative scopes assigned to quantifiers, and this will often change the claim made by a sentence. We saw the examples in 7.1.1 where such changes corresponded to different possible interpretations of ambiguous sentences. Ambiguity is less pronounced with the limited range of quantifier phrases we are dealing with in this chapter, so certain ways of choosing the order of analysis will be definitely wrong.

One example where two interpretations do seem to be possible is the sentence *Only teenagers went to each showing*. As in the examples of 7.1.1, the two interpretations can be brought out by applying subject-predicate expansion in two different ways:

Only teenagers are such that (they went to each showing)

Each showing is such that (only teenagers went to it)

The first says that, if you can find people who went back for each showing, they are all teenagers while the second says that the audience at each showing (if there was any) consisted solely of teenagers. Unlike the ambiguous examples of 7.1.1, neither of these claims implies the other.

The corresponding two analyses are the following:

Only teenagers are such that (they went to each showing)

$(\forall x: \neg Tx) \neg x \text{ went to each showing}$

$(\forall x: \neg Tx) \neg \text{each showing is such that } (x \text{ went to it})$

$(\forall x: \neg Tx) \neg (\forall y: Sy) x \text{ went to } y$

$(\forall x: \neg Tx) \neg (\forall y: Sy) Wxy$

$\forall x (\neg Tx \rightarrow \neg \forall y (Sy \rightarrow Wxy))$

Each showing is such that (only teenagers went to it)

$(\forall y: Sy) \text{only teenagers went to } y$

$(\forall y: Sy) \text{only teenagers are such that } (they \text{ went to } y)$

$(\forall y: Sy) (\forall x: \neg Tx) \neg x \text{ went to } y$

$(\forall y: Sy) (\forall x: \neg Tx) \neg Wxy$

$\forall y (Sy \rightarrow \forall x (\neg Tx \rightarrow \neg Wxy))$

[S: $\lambda x (x \text{ is a showing})$; T: $\lambda x (x \text{ is a teenager})$; W: $\lambda xy (x \text{ went to } y)$]

The first denies the generalization $x \text{ went to each showing}$ in

any case where x is a not a teenager. The second says of each showing that non-teenagers stayed away.

In other cases, there is less room for alternative interpretations. Since two of the kinds of generalization we are considering are negative, decisions about the relative scope of quantifier phrases are often at the same time decisions about the relative scope of negations and quantifier phrases, and English tends to be more unambiguous in that regard. We saw in 7.3.3 that the word *any* can be used to indicate that a sentence containing negation is not the denial of a generalization but rather the assertion of a generalization whose attribute is negative. For example, compare *No one saw everything* with *No one saw anything*. The first says of each person, x, that the generalization x *saw everything* is false while the second asserts of each thing that no one saw it. That is, the proper analyses of the two are the following:

$$\begin{aligned}
 & \textit{No one saw everything} \\
 & \textit{No one is such that (he or she saw everything)} \\
 & (\forall x: x \textit{ is a person}) \neg x \textit{ saw everything} \\
 & (\forall x: Px) \neg \textit{everything is such that (x saw it)} \\
 & (\forall x: Px) \neg \forall y x \textit{ saw } y \\
 & (\forall x: Px) \neg \forall y Sxy \\
 & \forall x (Px \rightarrow \neg \forall y Sxy) \\
 & \textit{No one saw anything} \\
 & \textit{Everything is such that (no one saw it)} \\
 & \forall y \textit{ no one saw } y \\
 & \forall y \textit{ no one is such that (he or she saw y)} \\
 & \forall y (\forall x: x \textit{ is a person}) \neg x \textit{ saw } y \\
 & \forall y (\forall x: Px) \neg Sxy \\
 & \forall y \forall x (Px \rightarrow \neg Sxy)
 \end{aligned}$$

[P: $\lambda x (x \textit{ is a person})$; S: $\lambda xy (x \textit{ saw } y)$]

The first of these sentences is perhaps slightly ambiguous (with an outside chance that it would be interpreted in the way indicated by the second analysis) but the second is pretty clearly unambiguous. It should be added that this is in part a consequence of the choice of verb and tense; the sentence *No one will eat anything* could perhaps be understood in the way indicated by the first analysis—that is, with *no one* as the main quantifier phrase—and *No one will eat just anything* has that as its most natural

interpretation.

A second pair of examples involves the other sort of negative generalization.

Only experts recognized every name on the list

Only experts are such that (they recognized every name on the list)

$(\forall x: \neg x \text{ is an expert}) \rightarrow x \text{ recognized every name on the list}$

$(\forall x: \neg x \text{ is an expert}) \rightarrow \text{every name on the list is such that } (x \text{ recognized it})$

$(\forall x: \neg Ex) \rightarrow (\forall y: y \text{ is a name on the list}) x \text{ recognized } y$

$(\forall x: \neg Ex) \rightarrow (\forall y: y \text{ is a name} \wedge y \text{ is on the list}) Rxy$

$(\forall x: \neg Ex) \rightarrow (\forall y: Ny \wedge Oyl) Rxy$

$\forall x (\neg Ex \rightarrow \neg \forall y ((Ny \wedge Oyl) \rightarrow Rxy))$

Only experts recognized any names on the list

Every name on the list is such that (only experts recognized it)

$(\forall y: y \text{ is a name on the list}) \text{ only experts recognized } y$

$(\forall y: y \text{ is a name on the list}) \text{ only experts are such that } (they \text{ recognized } y)$

$(\forall y: y \text{ is a name} \wedge y \text{ is on the list}) (\forall x: \neg x \text{ is an expert}) \rightarrow x \text{ recognized } y$

$(\forall y: Ny \wedge Oyl) (\forall x: \neg Ex) \rightarrow Rxy$

$\forall y ((Ny \wedge Oyl) \rightarrow \forall x (\neg Ex \rightarrow \neg Rxy))$

[E: $\lambda x (x \text{ is an expert})$; N: $\lambda x (x \text{ is a name})$; O: $\lambda xy (x \text{ is on } y)$; R: $\lambda xy (x \text{ recognized } y)$; l: *the list*]

Again, though there may be some hint of ambiguity, the interpretations represented by these analyses are by far the most likely ones. However, restating the second sentence as *Only experts recognized any name on the list* might increase the chance that it would be understood as equivalent with the first.

Another example shows that the use of *any* occurs not only with negative generalizations but also in the restricting predicates of affirmative generalizations.

Everything that is relevant to everything is worth knowing
Everything that is relevant to everything is such that (it is worth knowing)

$(\forall x: x \text{ is relevant to everything}) x \text{ is worth knowing}$
 $(\forall x: \text{everything is such that } (x \text{ is relevant to it})) x \text{ is worth knowing}$
 $(\forall x: \forall y x \text{ is relevant to } y) Wx$
 $(\forall x: \forall y Rxy) Wx$
 $\forall x (\forall y Rxy \rightarrow Wx)$

Everything that is relevant to anything is worth knowing
Everything is such that (everything that is relevant to it is worth knowing)

$\forall y \text{ everything that is relevant to } y \text{ is worth knowing}$
 $\forall y \text{ everything that is relevant to } y \text{ is such that (it is worth knowing)}$
 $\forall y (\forall x: x \text{ is relevant to } y) x \text{ is worth knowing}$

$\forall y (\forall x: Rxy) Wx$
 $\forall y \forall x (Rxy \rightarrow Wx)$

[R: $\lambda xy (x \text{ is relevant to } y)$; W: $\lambda x (x \text{ is worth knowing})$]

Notice that we could reverse the order of $\forall y$ and $\forall x$ in the statement of the second analysis with unrestricted quantifiers. That would trace the difference between it and the corresponding way of writing the first analysis to the location of $\forall y$ in relation to the parentheses. The difference in meaning between the these two sentences should make it clear that the placement of parentheses is as important in the case of quantifiers as it is in the case of connectives.

The moral to be drawn from the last three pairs of examples is to watch for cases where there are several quantifier phrases indicating generalization and one of them uses the word *any* or uses the word *every* in such a way that replacing it by *any* would change the meaning. As a rule, in cases where *any* and *every* contrast with one another, the word *any* indicates that the quantifier phrase has wider relative scope than some other operation (either a connective or a quantifier) and should be analyzed before this other operation while the word *every* indicates narrower scope than this other operation. There are many possibilities for “other operation” mentioned here. Negation and negative generalization are probably the most common, but we

have seen examples also of a contrast between *any* and *every* occurring in the antecedents of conditionals and in the restrictions of affirmative generalizations. When the other operation is one that we do not capture in our analyses, we will be able to identify the generalization only in the sentence in which it has wide scope. For example, we can analyze *It might affect anyone* by way of *Everyone is such that (it might affect him or her)* but we cannot analyze *It might affect everyone* without seeing it as the result of applying an operation marked by the modal auxiliary *might* to a generalization.

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