

7.3.1. Generalizations and counterexamples

So far we have concentrated on quantifier phrases that are used to state generalizations, but there are others that are used to deny them. For example, *Not every dog barks* denies that the attribute of barking holds universally for the domain of dogs, and the phrase *not every dog* may always be used to deny affirmative generalizations concerning dogs. Another example of a denied generalization is *Not only trucks were advertised*, and the phrase *not only trucks* serves to deny negative generalizations concerning domains complementary to the class of trucks.

If we remove the common noun phrases from these quantifier phrases, we are left with the phrases *not every* and *not only*. These have a function comparable (though opposed) to that of the quantifier words *every* and *only*. Thus we might think of *not every* and *not only* signs indicating the kind of generalization being denied and develop an approach to the analysis of sentences containing them that is parallel to the one we developed for kinds of generalization. However, it is easier to deal with these sentences by separating the word *not* off from *every* and *only* as a sign for negation (though it is worth noting that *not* functions in the non-standard way in such cases since it does not modify the main verb). So we can regard the sentences first as truth functional compounds and later analyze their negated components as generalizations. Here are the analyses

Not every dog barks

\neg *every dog barks*

$\neg (\forall x: x \text{ is a dog}) x \text{ barks}$

$\neg \forall x (x \text{ is a dog} \rightarrow x \text{ barks})$

Not only trucks were advertised

\neg *only trucks were advertised*

$\neg (\forall x: \neg x \text{ is a truck}) \neg x \text{ was advertised}$

$\neg \forall x (\neg x \text{ is a truck} \rightarrow \neg x \text{ was advertised})$

Negations definitely pile up in the second. There are equivalent forms with fewer (think, for example, of the analysis of *Not everything that was advertised was a truck*) but such sentences miss some of the indirection of the original English.

Although we have extracted the word *not* from the units *not*

every and *not only* in these analyses, it is grammatically a part of them. We might ask if there is a word or phrase that plays a role analogous to *not every* and *not only* in the case of negative direct generalizations. We can see more clearly the sort of expression this would be by recalling that to deny a generalization is to claim the existence of a counterexample to it. For example, *Not every dog barks* claims the existence of a dog that does not bark; and *Not only new listings were distributed* claims the existence of something other than a new listing that was nonetheless distributed. So, to find an expression that can be used to deny a negative direct generalization, we should look for an expression that can be used to claim the existence of counterexamples to such generalizations. Take the example *No dog climbs trees*. This says that the attribute of not climbing trees holds universally for dogs. A counterexample to such a claim would be a dog that does climb trees, so we are looking for a way of claiming that such a counterexample exists. English has an especially rich supply of ways of doing this, among them are *Some dog climbs trees*, *There is a dog that climbs trees*, and simply *A dog climbs trees*. None of these contain the word *not*, but they are still contradictory to generalizations and thus may be expressed as negations of those generalizations.

We might then treat a sentence using such an expression as negative and carry out the following sort of analysis:

$$\begin{aligned}
 & \textit{Some dog climbs trees} \\
 & \neg \textit{no dog climbs trees} \\
 & \neg (\forall x: x \textit{ is a dog}) \neg x \textit{ climbs trees} \\
 & \neg \forall x (x \textit{ is a dog} \rightarrow \neg x \textit{ climbs trees}).
 \end{aligned}$$

Although it is important to note that this analysis is possible, we will not have much occasion to employ it because we will introduce a more direct symbolic representation of claims like this in the next chapter. This is entirely analogous to something in truth-functional logic. We can express any disjunction ϕ *or* ψ using only conjunction and negation—as $\neg(\neg\phi \wedge \neg\psi)$ —but the sign \vee provides a more perspicuous representation.

The indefinite article *a* is one device used to make the claims that can be analyzed in this way; but, oddly, it also can be used to state direct affirmative generalizations. For example, *Every dog*

barks could be restated as *A dog barks*. Thus *A dog barks* is ambiguous and might be interpreted as either *Some dog barks* or *Every dog barks*. The use of *a* to state generalizations occurs only in a certain grammatical contexts, and there would be no ambiguity in *A dog climbed trees*, which cannot be understood to state a generalization. But, however special in form, generalizations stated using the indefinite article are quite common in use and, since we will be studying this use of the indefinite article first, you may need to remind yourself that the indefinite article is also used to claim the existence of examples.

It may seem strange that the same word should have acquired two such different roles; but, from one perspective, they have something in common. In both uses, the indefinite article can be taken as a sign that a free choice may be made. The two uses differ in whether this choice lies with the speaker or the audience. If I state the generalization *A dog likes bones*, I claim that, no matter what dog *you* pick, you will have something that likes bones. On the other hand, if I assert *A dog was digging in the garden*, I do not give you leave to choose any dog you please but claim only that it would be possible for *me* to pick a dog that was gardening.

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