7. Generalizations

7.1. Generalizations in English

7.1.0. Overview

Although quantifier phrases are grammatically similar to individual terms, their logical function is quite different; indeed, they are to predicates what predicates are to individual terms.

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7.1.1. Theories of quantifier phrases

In 6.2.1 individual terms like *Spot* and *the dog* were distinguished from other singular noun phrases like *every dog*, *no dog*, and *a dog*, whose semantic function is not to make definite references to single objects. We call phrases of the latter sort **quantifier phrases** and we will call words like *every*, *no*, and *a* that are used to form them **quantifier words**. The term *quantifier* here is intended to suggest the semantic function of these phrases: they serve to say how many objects of a certain kind have a certain property. Thus *Every dog likes bones* says that all objects of the kind indicated by the term *dog* have the property expressed by the predicate λx (x *likes bones*) while *No dog climbs trees* says that 0 objects of this kind have the property of climbing trees. Sentences with an indefinite article—for example, *A dog has been digging in the garden*—are particularly instructive. The claim made by this sentence is that at least one dog has the property of having been digging in the garden. (Or else that there *was* at least one dog that *had* the property of digging in the garden. The difference between the two interpretations is important but is one we cannot explore in this course. In the first we regard the tense of the sentence as part of the predicate while in the second we understand it to apply to the whole sentence. The second interpretation would preclude further analysis without some means of representing tense, something we will not explore.) Although the truth of this sentence can, in some cases, be traced to the activities of an individual dog it is not a claim about any individual dog but rather about the class of dogs as a whole. That is why its denial, *No dog has been digging in the garden*, does not give even the appearance of concerning an individual dog.

The study of quantifier phrases is one of the oldest parts of deductive logic but it is also the part that was last to be fully developed. The study of sentences like the examples above was the heart of Aristotle's work in logic and it is the one part of logic that was preserved in periods (such as the Renaissance and early modern era) when interest in logic dwindled and much of truth-functional logic was ignored or forgotten. However, Aristotle's account of these sentences was not fully satisfactory, and the problems they raised were not solved completely until the work of Frege.

Aristotle's treatment of quantifier phrases is the basis of the traditional theory of **syllogisms**. That theory was principally concerned with the sentences that contained quantifier phrases and it applied directly only to sentences that could be restated with a single quantifier phrase in subject position. It could not account for the validity of arguments that depended on the presence and interaction of several quantifier phrases in the same sentence. As an example of the latter, a traditional one, notice that *Every horse is a mammal* implies *Any head of a horse is a head of a mammal*, an implication that turns on the function of the two quantifier phrases *a horse* and *a mammal* in the conclusion.

Medieval logicians began to develop an account of quantifier phrases that

looked at individual phrases rather than at a limited variety of sentences containing them. The idea was to look at quantifier phrases, like individual terms, as serving to refer—with individual terms and the various quantifier phrases referring in different ways. The sort of reference, or **supposition**, would depend, in the case of quantifier phrases, on the quantifier word used and also on the place of the quantifier phrase in the sentence. Thus we might speak, as indeed we have, of *Spot* and *the dog* as making a definite reference. And, in a similar way, we might speak of *every dog* and *no dog* as both making a general reference to dogs (differing in whether they say they all have or all lack a given property) and of *a dog* as making an indefinite reference to a dog (since no more than one dog need be in question but no particular dog is crucial). This, by itself, does not settle the problem of multiple quantifier phrases but it does provide an approach to quantifier phrases that could be applied to each of several phrases in the same sentence.

The problem with a theory like this lies in the possibility that the kind of reference a quantifier phrase exhibits depends not only on the quantifier word it contains and its place in a sentence but also on its relation to other quantifier phrases. The ambiguity of sentences like the following one shows that the interaction of quantifier phrases can be important:

A reporter interviewed each juror.

This could be understood to assert either the weak claim that each juror was interviewed or the stronger claim that there was a single reporter who conducted each interview. The difference between these two claims can be brought out by restating the sentence to make one or the other of the quantifier phrases the subject followed by the phrase *is such that*. We will call this way of rephrasing a sentence a **subject-predicate** expansion.

weak reading: **Each juror** *is such that (***a reporter*** interviewed him or her)*

strong reading: **A reporter** *is such that (he or she interviewed* **each juror***)*

If we were to account for this ambiguity by something like a theory of supposition, we would have to find some sort of ambiguity in the quantifier phrases. The most natural way of doing this would be to say that a phrase like *a reporter* can have two sorts of reference, either a fixed indefinite reference or a variably indefinite reference (giving rise to the strong and weak readings, respectively); and this is roughly the account given by medieval logicians working on the theory of supposition.

The difficulty is that this sort of ambiguity is not the only one that can arise. For example, consider the following:

Every reporter asked a question of each juror

This is four ways ambiguous, with the interpretations indicated in the following table:

weakest reading: **Every reporter** *and* **juror** *are such that the former asked the latter* **a question**

intermediate reading 1: *For* **every reporter**, **some question** *is such that he or she asked it of* **each juror**

intermediate reading 2: *For* **each juror**, **some question** *is such that* **every reporter** *asked it of him or her*

strongest reading: **A question** *is such that* **every reporter** *asked it of* **each juror**

The strongest reading entails all the rest and all entail the weakest reading, but neither of the two intermediate readings entails the other.

The problem that this example poses for a theory of supposition is not simply that the different meanings of *a question* have begun to mount up implausibly but that the differences cannot be described without reference to other quantifier phrases. That is, *a question* could be said to exhibit a variably indefinite reference not only in the weakest reading but also in the two intermediate readings. These three readings differ because the variation in reference is allowed to depend on different factors: on both the reporter and the juror (the weakest), on the reporter only (the first intermediate reading), or on the juror only (the second intermediate reading). And this makes it seem that the differences lie not in the way a single quantifier phrase refers but in the way the sentence as a whole is put together.

One way of diagnosing the problem is to note that, while the theory of supposition provides the resources for a more subtle description of the ways sentences can be put together than is available in Aristotelian logic, it does not provide a way of describing the structure of sentences in varying levels of detail comparable to the gradually developing analyses that we can offer using connectives. There were hints of another approach in the Middle Ages but nothing was worked out fully until Frege's *Begriffschrift* (or 'Concept notation') of 1879. And it is hard to see how anything like Frege's solution to the problem of quantifier phrases could have been developed much earlier; for it depended on his understanding of predicates as expressing truth-valued functions of objects. This idea was a considerable extension of the concept of a function current in his day, and this had already been extended far beyond anything that might have occurred to a mathematician in the 18th century, to say nothing of a philosopher in the Middle Ages.

Still the medieval hints and Frege's final approach handled the problem we have been considering in a similar way. In both views, predicates come first and quantifier phrases are added to them—and added to them in a certain order. The different readings we may give to the resulting sentence then depend on the order in which the quantifier phrases were added. In the case of *Every reporter asked a question of each juror*, the strongest reading comes when *a question* is added last, for then we are saying that at least one question has the property expressed by λx (*every reporter asked* x *of every juror*). The weakest reading comes when we add this quantifier phrase first, for then we

say only that the predicate λxy (x *asked a question of* y)—roughly, λxy (x *questioned* y)—is true of every reporter and juror. The other two possibilities come when *a question* has been added to the sentence before one but not the other of the phrases *every reporter* and *each juror*.

What distinguished Frege's approach was that he went beyond the metaphor of quantifier phrases lining up to be added to a sentence. Instead, he spoke of quantifier phrases as signs for operations that apply to predicates just as predicates are operations that apply to individual terms. Thus, on this view, the grammatical similarity between individual terms and quantifier phrases disguises the semantic difference that is shown in Figure 7.1.1-1:
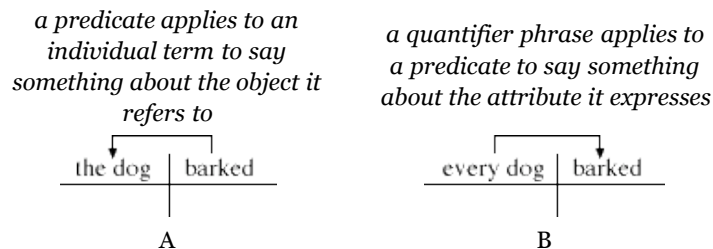

Fig. 7.1.1-1. The semantic roles of an individual term (A) and a quantifier phrase (B).

This kind of analysis can be applied to a sentence several times over by using the device of subject-predicate expansion. For example, the second intermediate reading of the quadruply ambiguous sentence above could be expressed along with its component predicates as follows:

Each juror is such that (a question is such that (every reporter asked it of him or her))

a question is such that (every reporter asked it of _____)

every reporter asked of _____

Here each quantifier phrase is seen to apply to a predicate that may contain previously applied quantifier phrases.

7.1.2. Pronouns and quantifier phrases

When a logically complex predicate is applied to an individual term, the result can be restated as a compound of simple predications. We have used this as basis for analyses like that of *That car is cheap and reliable* into *That car is cheap* and *That car is reliable*. However, when a sentence is formed from a complex predicate by applying a quantifier phrase to this predicate, no such simplification need be possible. The sentence *Some company is such that it does business in Tokyo and Terre Haute* cannot be restated as the conjunction of *Some company does business in Tokyo* and *Some company does business in Terre Haute*.

The difference between the latter case and the case of individual terms lies in the different relation between the logical form of the complex predicate and the logical form of the sentence as a whole depending on whether the predicate is applied to an individual term or has a quantifier phrase applied to it. The logical form of a complex predicate—as it would be displayed were we to analyze the body of a predicate abstract—reflects the form of any sentence that results from predicating it of a term. So every predication of this predicate has the same sort of structure. For example, if we apply the predicate λx (x *does business in Tokyo and Terre Haute*) to a term—*IBM* say—we will be asserting a conjunction because the predicate is a conjunction with blanks.

On the other hand, the claim made when a complex predicate has a quantifier applied to it is a claim made about the population of the predicate's extension, and there is no reason to think that the form of this claim will reflect the logical form of the predicate's claims about individual objects. In the example at hand, *Some company is such that it does business in Tokyo and Terre Haute* says that the extension of λx (x *does business in Tokyo and Terre Haute*) contains at least one company. To say this is to say something about the way in which the extensions of λx (x *does business in Tokyo*) and λx (x *does business in Terre Haute*) overlap and this sort of relation between their extensions cannot be expressed by a truth functional compound of claims made about the two extensions individually. This is one instance of a general point: few interesting relations between things can be restated as truth-functional compounds of claims made about the things

individually. For example, try restating the claim *John Stuart Mill was the son of James Mill* as a truth-functional compound of claims made about J. S. Mill and his father, each in isolation.

This difference between individual terms and quantifier phrases has an impact on the significance of pronouns whose antecedents are quantifier phrases. A pronoun that has an individual term as an antecedent is a device for avoiding repetition and can be eliminated if we are willing to tolerate the repetition. That is why nothing like pronouns was introduced into our symbolic notation for truth-functional logic. We are able to restate *Jack built the house and sold it* so that the pronoun *it* is replaced by a second occurrence of *the house*, giving us a compound of two independent components. But no such restatement is possible with *Jack built **a** house and sold it* (since *Jack built a house and sold a house* does not say he sold a house he built) so we cannot eliminate the pronoun.

Something similar can happen with compounded predicates and other compounded phrases. We can restate the sentence *The Titanic ran into an iceberg and sank* so that *and* joins clauses rather than predicates if we repeat the phrase *the Titanic*. But *A ship ran into an iceberg and sank* cannot be restated as a conjunction of clauses by repeating the phrase *a ship*. Since we regard conjunction only as an operation on sentences, a restatement as a conjunction of clauses is necessary if we are to subject the predicate λx (x *ran into an iceberg and sank*) to further analysis. And, because we cannot repeat *a ship* without changing the meaning, we must introduce a pronoun with *a ship* as its antecedent. So not only are we often prevented from eliminating pronouns with quantifier phrase antecedents, we are often forced to introduce such pronouns in order to analyze sentences.

7.1.3. Finding quantifier phrases

The examples of quantifier phrases we have been considering take an especially simple form: a quantifier word modifying a common noun. And these two elements form the core of most quantifier phrases. (Not all, because a sentence like *Lions are carnivores* achieves the same effect as the generalization *Every lion is a carnivore* by using the plural rather than a quantifier word.) Words and phrases other than quantifier words can appear as further modifiers of the common noun in a quantifier phrase; in particular, adjectives, relative clauses, and adjectival prepositional phrases often serve this function. For example, in

Every large dog in the neighborhood that was outside last night was barking

the underlined quantifier phrase is the bulk of the sentence. Besides the common noun *dog* and quantifier word *every*, it employs the adjective *large*, the prepositional phrase *in the neighborhood* and the relative clause *that was outside last night* to further specify the claim that is being made.

We encountered a same array of possible modifiers in the case of definite descriptions and the problem of finding the whole of a definite description has an analogue in the case of quantifier phrases. That is, when locating quantifier phrases, we must be on the lookout for modifiers that are part of the phrase. As was the case with definite descriptions, prepositional phrases and relative clauses are especially problematic here since they appear after the common noun. One test that was suggested for definite descriptions is particularly important in the case of quantifier phrases try replacing the phrase you have isolated by the pronoun *it*. Since this pronoun will not accept the modifiers that might be left behind, the result will be ungrammatical if modifiers have been left. Thus the first sentence below is grammatical, but the second and third are not.

It was barking
* *It that was outside last night was barking*
* *It in the neighborhood that was outside last night was barking*

Since we will make such replacements by pronouns as part of the analysis of a quantifier phrase, this test will be performed as a matter of course.

Of course, some contexts will require *he*, *she*, or the like, and the test will then be less clear-cut since these pronouns can accept modifiers. What you need to check is whether the pronoun is able to have an earlier antecedent. For example, in *Everyone who hesitates is lost*, replacing *everyone* by *he* gives us the sentence *He who hesitates is lost*, which is not only grammatical but has the same meaning as the original (when *he* is understood generically). But we cannot regard *he* in this context as having an earlier antecedent. In *Sam is indecisive and he who hesitates is lost*, the pronoun *he* must still be used to make a general claim and cannot refer specifically to Sam.

It is also important to avoid slipping between restrictive and non-restrictive relative clauses. For example, *Sam was indecisive and he, who hesitated, was lost* comes close to being grammatical and *he* has *Sam* as its antecedent. But this only works because the relative clause *who hesitated* is non-restrictive here (note the commas), and it is restrictive in *Everyone who hesitated was lost*.

Prepositional phrases present a special problem when quantifier phrases are in predicates since prepositional phrases can have adverbial as well as adjectival roles and a prepositional phrase left behind in the predicate when a quantifier phrase is analyzed need not make the sentence ungrammatical. Sentences can even be ambiguous in this respect. For example, *Larry heard of a new band in Indianapolis* might speak of Indianapolis either as the home of the band or as the place where Larry learned of them. The difference is captured by the following subject-predicate expansions:

> *A new band in Indianapolis is such that (Larry heard of it)*
> *A new band is such that (Larry heard of it in Indianapolis)*

The difference can be brought out also by another test suggested in the case of definite descriptions: converting the prepositional phrase into a relative clause. (In the example above, the clause would be *that is in Indianapolis* or *that was in Indianapolis*.) Doing so will force the prepositional phrase to be understood as adjectival and thus show the effect of treating it as part of the quantifier phrase. On the other hand, moving a prepositional phrase to the beginning of the sentence will to force to be understood adverbially and thus as not part of the quantifier phrase. The results of these tests are shown for two sentences below.

Diane studied a stellar object <u>at the edge of the known universe</u>

expansions	*A stellar object <u>at the edge of the known universe</u> is such that (Diane studied it)*
	A stellar object is such that (Diane studied it <u>at the edge of the known universe</u>)
conversion to relative clause	*Diane studied a stellar object <u>that was at the edge of the known universe</u>*
movement to the front	*<u>At the edge of the known universe</u>, Diane studied a stellar object*

Diane studied a dinosaur <u>in her paleontology class</u>

expansions	*A dinosaur <u>in her paleontology class</u> is such that (Diane studied it)*
	A dinosaur is such that (Diane studied it <u>in her paleontology class</u>)
conversion to relative clause	*Diane studied a dinosaur <u>that was in her paleontology class</u>*
movement to the front	*<u>In her paleontology class</u>, Diane studied a dinosaur*

These should convince you that (outside of science fiction) the prepositional phrase in the first sentence is probably intended to be adjectival and part of the quantifier phrase while the prepositional phrase of the second is most likely to be adverbial and part of the predicate.

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7.1.4. Kinds of generalizations

All quantifier phrases serve to say something about the extensions of predicates—specifically, about the number of objects in these extensions. But this description applies more naturally to some quantifier phrases than to others. For example, to restate *Every dog likes bones* as a numerical claim, we must resort to *Zero dogs fail to like bones*. We could describe the role of *every dog* in *Every dog likes bones* more naturally by saying that it is used to state a **generalization** about the property of liking bones. All the quantifier phrases we will consider in this course could be described, again some more naturally than others, in terms of the making and denying of generalizations. For example, *A dog has been digging in the garden* could, in a pinch, be regarded as the denial of the negative generalization *No dog has been digging in the garden*. We will begin our study of quantifier phrases by looking at them in terms of the idea of generalization. This will make the properties of phrases like *every dog* and *no dog* stand out more clearly than those of phrases like *a dog*. So we will reserve full attention to the latter phrase and its kin for the next chapter, where we return to looking at quantifier phrases as ways of making more obviously numerical claims. For the time being, we will think of sentences like *A dog has been digging in the garden* as statements that claim the existence of falsifying examples, or **counterexamples**, to the generalizations on which we will focus our attention.

We will begin our study of generalizations by developing some terminology for describing them informally before offering symbolic representations. A generalization claims that a certain property holds of all objects in a certain collection. We will refer to the collection of objects over which the generalization is made as the **domain** of the generalization and refer to the property that is said to hold generally as the **attribute** of the generalization. The chief problem in analyzing generalizations will be to identify the domain and the attribute. In a simple case, like *Every dog likes bones*, the domain will be the class of objects picked out by the common noun of the quantifier phrase together with its modifiers (here it is the class of dogs), and the attribute will be the property expressed by the predicate to which the quantifier phrase is applied

(here it is the property of liking bones). We will refer to the common noun together with its modifiers as the **class indicator** of the generalization and the predicate to which the quantifier phrase is applied as the **quantified predicate**.

In the simple case of *Every dog likes bones* we would have

class indicator
↓
quantifier phrase → *Every* | *dog* | *likes bones*   ← quantified predicate
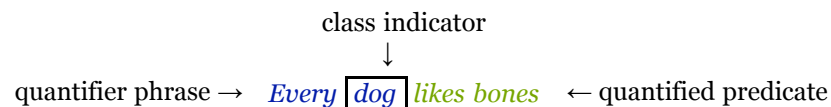
but with a more complex quantifier phrase we might have something like

Every | *large dog in the neighborhood that was outside last night* | *was barking*

with modifiers of the common noun included in the class indicator.

There are two common ways that the quantified predicate of a generalization is related to its attribute. When the attribute is the property expressed by the quantified predicate, we will say that the generalization is **affirmative**. So *Every dog barks* is an affirmative generalization since barking is both the attribute that is said to hold generally of dogs and the property expressed by the predicate λx (x *barks*). On the other hand, *No dog climbs trees* is not affirmative. The domain of this generalization is also the class of dogs but what is said to hold generally of them is that they do not climb trees. That is, the attribute of this generalization is the denial of the property expressed by the quantified predicate. When this is the relation between the quantified predicate and the attribute—i.e., when the attribute is expressed by the negation of the quantified predicate—we will say that the generalization is **negative**. Notice that this does not characterize the claim made by the generalization so much as the way this claim is expressed. The generalization *Every box was unopened* is affirmative because its attribute is the property of not being opened and this is the property expressed by the quantified predicate λx (x *was unopened*). On the other hand the generalization *No box was opened* is negative even though it makes the same claim.

There are also two common ways the domain of a generalization is related to the class indicator. A **direct generalization** is one whose domain is identical with the class of objects picked out by its class indicator; that is, it is identical with the **indicated class**.

Thus, both *Every dog barks* and *No dog climbs trees* are direct because in both cases the domain, dogs, is picked out directly by their class indicators. However, the generalization

<div align="center">

Only trucks were advertised

</div>

is not direct. Its domain is not the class of trucks, but the class of non-trucks, whose members are said not to have been advertised. To see this, think what the counterexamples to the generalization would be like: non-trucks that were advertised. We will refer to such generalizations as **complementary**. A complementary generalization will often have the corresponding direct generalization as an implicature; for example, *Only new listings were distributed* suggests that *all* new listings were distributed. But *Only trucks were advertised* carries no such implicature, and the implicature is easily canceled in other cases; the sentence *Only new listings were distributed and not even all of them were* is not at all incoherent, much less self-contradictory.

We use the term *complementary* for a generalization like *Only trucks were advertised* because its domain is the complement of the indicated class. The **complement of** a set X **relative to** a set Y is the class of all members of Y that are not in X (see Figure 7.1.4-1). It will sometimes be useful to turn this idea around and think of the complement of X relative to Y as Y **with** X **subtracted**. When only the class X is specified, the complement must be taken relative to the set of all reference values; this yields the **full complement of** a set, the class of every value not in the set. Thus a complementary generalization makes a claim about the full complement of the indicated class.
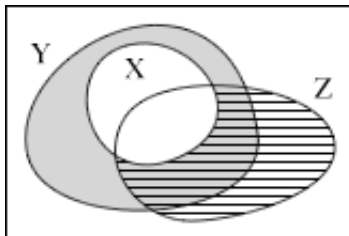

Fig. 7.1.4-1. The complements of a set X relative to sets Y (in gray) and Z (hatched).

The basic classification of generalizations we have considered is summarized in Table 7.1.4-1 below. Each entry shows an English form that can be used as a standard paraphrase for that sort of generalization.

	Affirmative (the attribute is the property expressed by the quantified predicate)	*Negative* (the attribute is the denial of the property expressed by the quantified predicate)
Direct (the domain is the indicated class)	*Every* C *is such that ...it...*	*No* C *is such that ...it...*
Complementary (the domain is the complement of the indicated class)		*Only* Cs *are such that ...they...*

Table 7.1.4-1. The classification of generalizations.

There seems to be no quantifier word that indicates an affirmative complementary generalization, but some of the modifying phrases we will consider next can be used to state an affirmative generalization about the complement of a given class.

7.1.5. Bounds and exceptions

In general, the class indicator and quantified predicate of the generalization will be our starting points for identifying the domain and attribute, and the quantifier word appearing in the quantifier phrase will be our chief guide in doing this. For example, the quantifier words *every* and *all* will lead us to presume that the domain and attribute can be determined in the simple way used above. However, this is only a presumption—as is shown by the following example:

> *Among members of the House, all Republicans except Midwesterners supported the bill*.

Here the class indicator is *Republicans* but the domain is the class of non-Midwestern Republicans in the House. One way to see this is to ask yourself what sort of thing would be a counterexample; here it would be a non-Midwestern Republican House member who did not support the bill. (You can often sharpen your sense of the content of a generalization by asking what a counterexample would have to be like. This will tell you something about the domain and attribute of the generalization because a counterexample must be something in the domain that fails to have the attribute.)

The same generalization would be stated by sentence *Every non-Midwestern Republican who was a member of the House supported the bill*, which builds the full specification of the domain into its class indicator. On the other hand, in the displayed example, our path from the class indicator to the domain has been inflected by the phrases *among members of the House* and *except Midwesterners*, which function as modifiers of the quantifier phrase. We will call phrases like these **bounds indicators** and **exception indicators**, respectively. Each will consist of a common noun phrase, usually in the plural, together with a word showing that the class picked out by this common noun phrase is a class bounding the generalization, a **bounding class**, or that it is a **class of exceptions**. The words *among* and *of* are often used to marks bounds, and *except* and *but* can serve to mark exceptions. Although *other than* functions much like *except* and *but*, phrases formed with it have the grammatical status of adjectives and can be regarded as modifiers of the common noun that are part of the

class indicator.

Our description of the domains of direct and complementary generalizations applies only in cases where the are no bounds or exceptions to the generalization. When there is a bounding class, the true domain is the region where the domain we described earlier overlaps, or **intersects**, the bounding class; it is the **intersection** of the two classes. For example, the direct generalization

> *Among members of the House, all Republicans supported the bill*

has as its domain the class of members of the House who are Republicans, and this is the intersection of the class of Republicans with the class of House members. The complementary generalization

> *Of GM products, only trucks were advertised*

has as its domain the class of GM products that are non-trucks, the intersection of the class of non-trucks with the class of GM products.

The bounding class need not appear as explicitly as it does in these examples. It may be supplied by the context of use rather than by an explicit bounds indication. For example, someone who asserts *Everyone was affected by the drought* probably has in mind only the population of a particular region. And it is not unusual for the class indicator of a generalization to indicate both the bounding class and the complemented class through the use of emphasis. As examples, read the following with emphasis on the underlined expressions:

> *Only cans of mushrooms <u>from that plant</u> were recalled*
> *Only cans <u>of mushrooms</u> from that plant were recalled*

In each case, the attribute is the property of *not* having been recalled. In the first, the domain consists of cans of mushrooms that are not from the plant in question. In the second, it consists of cans from the plant that are not cans of mushrooms. (To confirm this for yourself, think what counterexamples to the claims would be like.)

We can get from the indicators to the domains of these examples by taking the bounding class as the class picked out by the unemphasized part of the class indicator (e.g., *cans of mushrooms*

or *cans from that plant*, respectively). The emphasis serves to mark the further specifications that move us from the bounding class to the narrower class that is complemented relative to it. Thus the domain in the second case can be described as the class of all cans from the plant that are not cans of mushrooms from the plant. Notice that this can be described either as the intersection of the full complement of the indicated class (i.e., cans of mushrooms from the plant) and the bounding class (i.e., all cans from the plant) or as the complement of the indicated class relative to the bounding class (i.e., all cans from the plant). In general, the intersection of the complement of a set X with a set Y is the complement of X relative to Y. (You can use ┌Figure 7.1.4-1┐ to help you think about this.)

The same device can appear with direct generalizations but its effect is primarily on implicatures. The sentence

> All <u>Republicans</u> *in the House supported the bill*

ends up having the same domain as

> *All Republicans <u>in the House</u> supported the bill*

but, while the first suggests (and it is only a suggestion) that some Democrats in the House did not support the bill, the second suggests that support faltered among Republicans in the Senate. In fact, what we are seeing here is again the effect of emphasis on complementary generalizations but its effect is on the complementary generalizations that are often implicated by direct generalizations.

The domain of a generalization is also modified when there is a class of exceptions—though this happens only with direct generalizations. If we set aside the effect of a bounding class, the domain of a generalization with exceptions is the indicated class with the class of exceptions subtracted; that is, the domain consists of the members of the indicated class that are not in the class of exceptions. For example, the domain of the generalization

> *All Republicans except Midwesterners supported the bill*

is the class of Republicans with the class of Midwesterners subtracted; it is the class of non-Midwestern Republicans. In the presence of the bounds indicator *among members of the House*, as in the example beginning this subsection, the domain is restricted further, to the members of the House who are non-Midwestern Republicans. Both bounds and exceptions modify a direct generalization by narrowing the domain, and they do so from independent directions so their effects accumulate without interaction. In particular, there is no need for concern about the order in which these modifications are applied; bounding and then excepting comes to the same thing as excepting and then bounding.

Our only examples of complementary generalizations were negative because there seemed to be no quantifier word in English that indicates a complementary affirmative generalization. This gap is filled by a peculiar device: the class indicator can be dropped entirely when an exception indicator is present. For example, if there were no bounds provided by the context, the affirmative generalization *All but the hardiest plants suffered* would have as its domain the full referential range with the class of hardiest plants subtracted. This is the same thing as the complement of the class of hardiest plants, so we have an affirmative generalization whose domain is a complement (though not the complement of the indicated class since there is no class indicated). We are rarely prepared to generalize affirmatively about everything outside a given class, so we can expect to find a bounding class in most cases. Here, it probably would be the class of all plants—though that would depend on the context and emphasis can play a role, too. The sentence *All but Midwestern Republicans supported the bill* leaves its bounding class to the context while *All but <u>Midwestern</u> Republicans supported the bill* marks its bounding class as the class of Republicans (though the context might bound it further).

When the class indicator is negative, the need for bounds applies somewhat differently. The sentence *All but non-smokers are at risk* needs no bounds but someone who asserts *All non-smokers were relieved* probably intends to limit its domain to people.

In general, a bounding class can be shown in a paraphrase by prefixing a phrase of the form *Among* B*s*. The effect of this phrase is to limit the domain to the bounding class by intersecting this class with the class obtained from the other specifications of the domain. A class of exceptions can be shown by putting the phrase of the form *except* E*s* after the quantifier phrase. Its effect is to limit the domain by subtracting the class of exceptions from the

class obtained from other specifications. In order to use these modifications grammatically when paraphrasing direct generalizations, we must first put the quantifier phrase in the plural (e.g., *All* C*s* or *No* C*s*). Paraphrases like these will often be stilted but they will make it easy to represent the form of the generalization symbolically.

Glen Helman   25 Aug 2005

7.1.s. Summary

Not all singular noun phrases are individual terms. Many are quantifier phrases formed using quantifier words . These phrases are used to say how many objects of a certain kind have a certain property. The study of them dates to Aristotle's theory of syllogisms and was active again in medieval theories of " supposition " but some central problems were solved only with Frege's work a century ago. The problems he solved concern the role of multiple quantifier phrases in the same sentence. To describe this role, we must describe the contribution of individual phrases in a way that takes account of the interaction between phrases, something that can lead to multiple ambiguity. Frege's approach was to regard quantifier phrases as signs for operations that apply to predicates, with the interaction of phrases accounted for by the order in which these operations are applied. A particular order of application can usually be fixed in English (and different interpretations of a sentence are distinguished) by subject-predicate expansions using the phrase *is such that*.

Complex predicates can be avoided when they apply to individual terms and all such sentences can be stated in reduced form, but this is not always possible in the case of quantifier phrases because a quantifier phrase may be used to state a relation between predicates that cannot be analyzed as a compound of separate claims about these predicates. For the same reason, pronouns with quantifier phrase antecedents often cannot be replaced by their antecedents, and we may even have to introduce pronouns with quantifier phrase antecedents when we paraphrase compound phrases by compound clauses.

The core of a quantifier phrase consists of a quantifier word and a common noun, but the noun may be modified by adjectives, relative clauses, or prepositional phrases. You can test to see if you have isolated the whole of a quantifier phrase by seeing if it would be grammatical to replace it by the pronoun *it*. Prepositional phrases in the predicate can often be understood to modify either a noun or a verb (and can be missed by the pronoun test); they are part of the quantifier phrase if they can be restated as relative clauses.

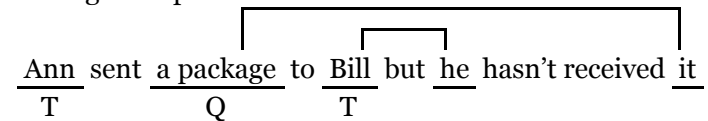A generalization claims that every object in a certain class, the

generalization's ⃞domain⃞ , has a certain property, the generalization's ⃞attribute⃞ . In the simplest cases the domain is the class (the ⃞indicated class⃞) that is picked out by the ⃞class indicator⃞ , the common noun plus modifiers of the quantifier phrase; and the attribute is expressed by the ⃞quantified predicate⃞ to which the quantifier phrase is applied. The content of a generalization can often be clarified by considering the sort of ⃞counterexample⃞ that would show it to be false. When the attribute of the generalization is expressed by the quantified predicate, the generalization is ⃞affirmative⃞ . The words *all* and *every* are used to express affirmative generalizations while *no* is used to express a ⃞negative⃞ generalization, one whose attribute is the denial of the quantified predicate. All these words express ⃞direct⃞ generalizations, whose domain is the indicated class; but, in other generalizations, ⃞complementary⃞ generalizations, the domain is outside the indicated class. In the simplest case, it is the full ⃞complement⃞ of this class, which is its ⃞complement relative⃞ to the class of all reference values—i.e., the result of ⃞subtracting⃞ it from this class.

   Other, less straightforward, relations between the wording of the generalization and the claim it makes can be indicated by modifiers of the quantifier phrase that we will label ⃞bounds indicators⃞ or ⃞exception indicators⃞ . The complement of an indicated class over which a generalization is made is often not its full complement but its complement relative to some ⃞bounding class⃞ , which may cited explicitly by a bounds indicator (such as *among* B*s*) or may be implicit in the context. There seems to be no word designed to express generalizations that are both affirmative and complementary. Nevertheless, direct generalizations may be restricted to objects outside a ⃞class of exceptions⃞ by use of such phrases as *except* E*s*. Bounds and exceptions limit the domain by ⃞intersecting⃞ it with another class (the bounding class or the complement of the class of exceptions); the order in which these operations are carried out makes no difference.

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⃞Glen Helman⃞ 25 Aug 2005

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7.1.x. Exercise questions

1. In the sentences below underline all individual terms and quantifier phrases, and any pronouns that have such expressions as antecedents. Distinguish individual terms and quantifier phrases by marking them T or Q and indicate what antecedents any pronouns are bound to—as in the following example:

    Ann sent a package to Bill but he hasn't received it
    T   Q   T

    a. *Ann saw a movie and told Bill about it*.
    b. *Tim watched a dance troop from India*.
    c. *If anyone backs out, they will get a refund*.
    d. *Dave called everyone he knew*.
    e. *Every dog in the kennel was barking*.
    f. *Bill heard something and Carol heard it, too*.
    g. *Tim watched a dance troop from the balcony*.

2. Check the sentences below for any ambiguity that may be traced to the order in which quantifier phrases have been applied; when more than one interpretation is possible, bring out the differences by means of subject-predicate expansion and indicate which of the interpretations imply which others.

    a. *Everyone works toward some goal*.
    b. *Each member of the committee read each application*.
    c. *Someone eats at a restaurant every day*.

3. For each of the following generalizations, take the following preliminary steps in its analysis: (i) separate the main quantifier phrase and quantified predicate through a restatement using *is such that* and mark the class indicator, (ii) describe the sort of thing that would count as a counterexample to the generalization, and (iii) determine whether the generalization is affirmative or negative and whether it is direct or complementary.

    a. *Every book was checked out*.

b. *Kathy spoke to each guest*.

c. *No one in the lobby had seen the package before the explosion*.

d. *Tod carefully noted everything he saw in the room*.

e. *No one who was familiar with both France and Germany was surprised*.

f. *The committee accepted only entries submitted before the deadline*.

4. The generalizations below have quantifier phrases that make essential use of emphasis (marked here by underlining) to indicate the bounds on the generalization. Restate them so that these bounds are explicitly indicated. For example, *Only patient children will complete the puzzle* could be paraphrased by *Among children, only patient ones will complete the puzzle*.

a. *Only new <u>commercial</u> vehicles are covered by the regulation*.

b. *Only French composers <u>of the early Baroque</u> used that device*.

c. *All but <u>emergency</u> vehicles were banned from the park*.

d. *Only <u>new</u> commercial vehicles are covered by the regulation*.

e. *Sam eats all but <u>orange</u> jelly beans*.

f. *Only <u>new commercial</u> vehicles are covered by the regulation*.

5. In each case below, use the information given about a generalization to determine its domain and attribute. Also, state such a generalization in English.

a. type: direct and affirmative
class indicator: *road*
quantified predicate: λx (x *is finished*)

b. type: direct and negative
class indicator: *road*
quantified predicate: λx (x *is finished*)

c. type: complementary and negative
class indicator: *road*
quantified predicate: λx (x *is finished*)

d. type: direct and affirmative
class indicator: *road*
quantified predicate: λx (x *is finished*)
class of exceptions: *urban freeways*

e. type: direct and negative
class indicator: *road*
quantified predicate: λx (x *is well-maintained*)
class of exceptions: *urban freeways*

f. type: direct and affirmative
class indicator: *roads*
quantified predicate: λx (x *is finished*)
bounding class: *federal projects*
class of exceptions: *urban freeways*

g. type: direct and negative
class indicator: *dentists*
quantified predicate: λx (x *frowned*)
bounding class: *alumni*
class of exceptions: *orthodontists*

7.1.xa. Exercise answers

1.

a.

Ann saw a movie and told Bill about it
T   Q   T

b.

Tim watched a dance troop from India
T   Q   T

c.

If anyone backs out, they will get a refund
Q   Q

d.

Dave called everyone he knew
T   Q

e.

Every dog in the kennel was barking
Q   T

f.

Bill heard something and Carol heard it , too
T   Q   T

g.

Tim watched a dance troop from the balcony
T   Q   T

2.

a. Two ways ambiguous:

   i) *Everyone is such that (he or she works toward some goal).*

   ii) *Some goal is such that (everyone works toward it).*

   (ii) implies (i).

b. Not ambiguous

c. Four ways ambiguous:

   i) *Every day is such that (someone eats at a restaurant on that day).*

   ii) *Someone is such that (every day is such that (he or she eats at a restaurant on that day)).*

   [Notice that, although *that day* is a demonstrative phrase (and might thus be counted an individual term), it does not have a reference independent of the quantifier phrase *every day*. It therefore functions here like a pronoun with *every day* as its antecedent. The

phrase *the truck* has a similar function in *A truck struck a car but only the truck was damaged*.]

   iii) *A restaurant is such that (every day is such that (someone eats at it on that day)).*

   iv) *Some person and restaurant are such that (he or she eats at it every day).*

   (iv) implies all the others; (ii) and (iii) each imply (i).

3. Class indicators are boxed.

a.

   i. *Every book was checked out.*
     Every `book` *is such that (it was checked out).*

   ii. *counterexample:* a book that was *not* checked out

   iii. *Direct and affirmative*.

b.

   i. *Kathy spoke to each guest.*
     Each `guest` *is such that (Kathy spoke to him or her).*

   ii. *counterexample:* a guest that Kathy *did not* speak to

   iii. *Direct and affirmative*.

c.

   i. *No one in the lobby had seen the package before the explosion.*
     No `one [i.e., person] in the lobby` *is such that (he or she had seen the package before the explosion).*

   ii. *counterexample:* a person in the lobby who *had* seen the package before the explosion

   iii. *Direct and negative*.

d.

   i. *Tod carefully noted everything he saw in the room.*
     Every `thing Tod saw in the room` *is such that (Tod carefully noted it).*

   ii. *counterexample:* something Tod saw in the room that he did *not* carefully note

   iii. *Direct and affirmative*.

e.   **i.** *No one who was familiar with both France and Germany was surprised.*
No one [i.e., person] who was familiar with both France and Germany *is such that (he or she was surprised).*

    ii. *counterexample:* someone familiar with both France and Germany who *was* surprised

    iii. *Direct and negative.*

f.   **i.** *The committee accepted only entries submitted before the deadline.*
Only entries submitted before the deadline *are such that (the committee accepted them).*

    ii. *counterexample:* an entry *not* submitted before the deadline that the commitee *did* accept [*Note:* this reflects the simplest interpretation of the generalization that does not make distinctions among parts of the class indicator; that is, it does not look for the differences in emphasis that are the topic of the next exercise.]

    iii. *Complementary and negative.*

4. **a.** *Among new vehicles, only commercial ones are covered by the regulation.*

   b. *Among French composers, only those of the early Baroque used that device.*

   c. *Among vehicles, all but emergency ones were banned from the park.*

   d. *Among commercial vehicles, only new ones are covered by the regulation.*

   e. *Among jelly beans, Sam eats all but orange ones.*

   f. *Among vehicles, only new commercial ones are covered by the regulation.*

5. **a.** *domain:* roads
attribute: the property of being finished
Every road is finished

   b. *domain:* roads
attribute: the property of being *un*finished
No road is finished

   c. *domain:* things that are not roads
attribute: the property of being *un*finished
Only roads are finished

   d. *domain:* roads that are not urban freeways
attribute: the property of being finished
All roads, except urban freeways, are finished

   e. *domain:* roads that are not urban freeways
attribute: the property of *not* being well maintained
No roads, except urban freeways, are well-maintained

   f. *domain:* federal projects that are roads but not urban freeways
attribute: the property of being finished
Among federal projects, all roads, except urban freeways, are finished

   g. *domain:* alumni who are dentists but not orthodontists
attribute: the property of *not* having frowned
Among alumni, no dentists, except orthodontists, frowned