6.1.2. Logical predicates

We derived the concept of an individual term from a traditional description of the grammatical subject of a sentence by focusing on the semantic idea of naming. As we will see in 6.2.1, the idea of an individual term is much narrower than the idea of a grammatical subject: not every phrase that could serve as the subject of a sentence counts as an individual term.

We have seen that the opposite is true of our concept of a predicate: it includes grammatical predicates but many other expressions, too. The definition of a predicate in our sense is, like the definition of an individual term, a semantic one: a predicate says something about the about whatever objects are named by the individual terms to which it is applied. The simplest example of this is a grammatical predicate that says something about an object named by an individual term. But consider a sentence that has not only a subject but also a direct object—*Ann met Bill* for example. This says something about Ann, but it also says something about Bill. From a logical point of view, we could equally well divide the sentence into the subject *Ann* on the one hand and the predicate *met Bill* on other or into the subject-plus-verb *Ann met* and the direct object *Bill*. And we will be most in the spirit of the idea that predicates are used to say something about individuals if we divide the sentence into the two individual terms *Ann* and *Bill* on the one hand and the verb *met* on the other. The subject and object both are names, and the verb says something about the people they name. That is why we define a ***predicate*** as an operation that forms a sentence when applied to one or more terms. We will speak of the application of this operation as ***predication*** and speak of a sentence that results as ***a predication***.

We can present predicates in this sense graphically by considering sentences containing any number of blanks. For example, the predication *Jane called Spot* might be depicted as follows:

Individual terms:	*Jane*		*Spot*
Predicate:	____	*called*	____

The number of different terms to which a predicate may be applied is its number of ***places***, so the predicate above has 2 places while predicates, like ____ *ran* and ____ *barked*, that are predicates in the grammatical sense will have one place.

In the example above, the two-place predicate is a transitive verb and the second individual term functions as a direct object in the resulting sentence. The individual terms that serve as input to predicates also often appear as indirect objects or as the objects of prepositional phrases that modify a verb—as in the following examples:

Individual terms: *Jane* *Spot* *the ball*
Predicate: ____ *threw* ____ ______

Individual terms: *the ball* *the window* *the fishbowl*
Predicate: ______ *went through* __________ *into* __________

Other examples of many-place predicates are provided by sentences containing comparative constructions or relative terms. Even conjoined subjects can indicate a many-place predicate when *and* is used to indicate the terms of a relation rather than to state a conjunction:

Individual terms: *Jane* *Sally*
Predicate: ____ *is older than* ____

Individual terms: *2* *5*
Predicate: _ < _

Individual terms: *Jane* *Sally*
Predicate: ____ *is a sister of* ____

Individual terms: *Jane* *Sally*
Predicate: ____ *and* ____ *are sisters*

Although you will rarely run into predicates with more than three or four places, it is not hard make up examples of predicates with arbitrarily large numbers of places. For example, imagine the predicate you would get by analyzing a sentence that begins *Sam travelled from New York to Los Angeles via Newark, Easton, Bethlehem, ...* and goes on to state the full itinerary of a trans-continental bus trip.

The places of a many-place predicate come in a particular order. For example, the sentences *Jane is older than Sally* and *Sally is older than Jane* are certainly not equivalent, so it matters which of *Jane* and *Sally* is in the first place and which in the second when we identify them as the inputs of the predicate ____ *is older than* ____ . Even when the result of reordering individual terms is equivalent to the original sentence, we will count the places as having a definite order and treat any reordering of the terms filling them as a different sentence. So *Dick is the same age as Jane* and *Jane is the same age as Dick* will count as different sentences even though ____ *is the same age as* ____ is symmetric in the sense that

$$\sigma \textit{ is the same age as } \tau \Leftrightarrow \tau \textit{ is the same age as } \sigma$$

for any terms σ and τ.

The only restriction on an analysis of a sentence into a predicate and individual terms is that the contribution of an individual term to the truth value of a sentence must lie only in its reference value—that is, only in what it names if it names something and only if in the fact that it names nothing if it does not and has the nil reference value mentioned in 1.3.4 . This means that the predicates we will consider are like truth-functional connectives in being ***extensional operations***: the extension of their output depends only on the extensions of their inputs.

In the specific case of predicates, this requirement is sometimes spoken of as a requirement of ***referential transparency***. When evaluating the truth-value of a sentence we sometimes look through individual terms and pay attention only to their reference values while in other cases we pay attention to the terms themselves or the ways in which they refer to their values because differences of this sort make a difference for the truth value of the sentence. For example, in deciding the truth of *The U. S. president is over 40* all that matters about the individual term *the U. S. president* is who it refers to. On the other hand, the sentence *For the past two centuries, the U. S. president has been over 35* is true while the sentence *For the past two centuries, George Bush has been over 35* is false—even when the terms *the U. S. president* and *George Bush* refer to the same person. So, in this second case, we must pay attention to differences between terms that have the same reference value. When this is so the occurrences of these terms are said to be ***referentially opaque***; that is, we cannot look through them to their reference values. The restriction on the analysis of sentences into predicates and individual terms is then that we can count an occurrence of an individual term as filling the place of a predicate only when that occurrence is referentially transparent. Occurrences that are referential opaque cannot be separated from the predicate and must remain part of it.

Hints of idea of a predicate as an incomplete expression can be found in the Middle Ages but it was first presented explicitly a little over a century ago by Gottlob Frege. Frege applied the idea of an incomplete expression not only to predicates but also to mathematical expressions for functions. Indeed, Frege spoke of predicates as signs for a kind of function, a function whose value is not a number but rather a truth value. That is, just as a function like addition takes numbers as input and issues a number as output, a predicate is a sign for a function that takes the possible references of individual terms as input and issues a truth value as output. It does this by saying something true or false about its input.

We will speak of the truth-valued function associated with a 1-place predicate as a ***property*** and speak of the function associated with a predicate of two or more places as a ***relation***. Occasionally, we will want to speak of properties and relations collectively; we will use the term ***attribute*** for this. Thus a predicate is a sign for an attribute in the way a truth-functional connective is a sign for a truth function.

Just as a truth-functional connective can be given a truth table, the extensionality of predicates means that a table can capture the way the truth values of the their output sentences depend on the reference values of their input. For example, consider the predicate ___ *divides* ___ *(evenly)*. Just as there can be addition or multiplication tables displaying the output of arithmetic functions for a limited range of input, we can give a table indicating the output of the relation expressed by this predicate. For the first half dozen positive integers, we would have the table shown below. Here the input for the first place of the predicate is shown by the row labels at the left and the input for the second place by the column labels at the top. The first row of the table then shows that 1 divides all six integers evenly, the second row shows that 2 divides only 2, 4, and 6 evenly, and the final column shows that each of 1, 2, 3, and 6 divides 6 evenly.

divides	1	2	3	4	5	6
1	T	T	T	T	T	T
2	F	T	F	T	F	T
3	F	F	T	F	F	T
4	F	F	F	T	F	F
5	F	F	F	F	T	F
6	F	F	F	F	F	T

Of course, this table does not give a complete account of the meaning of the predicate; and, for many predicates, no finite table could. But such tables like this will still be of interest to us because we will consider cases where there are a limited number of reference values and, in such cases, tables can give full accounts of predicates.

As was noted in 1.3.4, we assume that sentences have truth values even when they contain terms that do not refer to anything. This means that we must assume that predicates yield a truth value as output even the nil value is part of their input; that is, we assume that predicates are ***total***. The truth value that is issued as output when the input includes the nil value is usually not settled by the ordinary meaning of an English predicate. It is analogous to the supplements to contexts of use suggested in 1.3.2 as a way of handling cases of vagueness. As in that case, we try to avoid making anything depend on the particular output in cases of undefined input but instead look at relations among sentences that hold no matter how such output is stipulated.