5.4.s. Summary

The law for the conditional as a premise applies only to *reductio* arguments and provides a way of rejecting a conditional by deriving its antecedent φ from the premises and reducing its consequent to absurdity given the premises. The corresponding derivation rule is Rejecting a Conditional (RC).

This rule reflects the fact that a conditional is false when its antecedent is true and its consequent is false. The rules of Weakening (Wk) that have conditionals as conclusions reflect the fact that a conditional is true if its consequent is and also if its antecedent is false.

With these rules, the system of derivations for truth-functional logic is complete. It consists of the fundamental rules for developing gaps by exploiting resources or planning for goals, two rules each for negations, conjunctions, disjunctions, and conditionals along with a rule to plan for atomic sentences. There are the same four rules for closing gaps we had as of 3.2, and we now also have a set of four detachment rules that provide alternative ways of exploiting weak truth-functional compounds. These rules form the basic system; and all are progressive. In addition, there is a group of rules that are not necessarily progressive although they are sound and safe—the attachment rules and the general rule LFR for introducing lemmas in *reductio* arguments.

Rules for developing gaps				*Rules for closing gaps*		
	for resources	*for goals*			*when to close*	*rule*
atomic sentence		IP			the goal is also a resource	QED
negation ¬ φ	CR (if φ is not atomic and the goal is ⊥)	RAA			sentences φ and ¬ φ are resources & the goal is ⊥	Nc
conjunction φ ∧ ψ	Ext	Cnj			⊤ is the goal	ENV
disjunction φ ∨ ψ	PC	PE			⊥ is a resource	EFQ
conditional φ → ψ	RC (if the goal is ⊥)	CP				

Basic system

Detachment rules (optional)				*Attachment rules*		*Added rules (optional)*
main resource	*auxiliary resource*	*rule*		*added resource*	*rule*	
	φ	MPP		φ ∧ ψ	Adj	
φ → ψ	ψ̄	MTT		φ → ψ	Wk	
φ ∨ ψ	φ̄ or ψ̄	MTP		φ ∨ ψ	Wk	
¬ (φ ∧ ψ)	φ or ψ	MPT		¬ (φ ∧ ψ)	Wk	
				Rule for lemmas		
				prerequisite	*rule*	
				the goal is ⊥	LFR	

As in the earlier tables of this form, the names of the rules are links to places where they are actually stated.

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