5.4.2. Optional extras

The law for the conditional as a premise directly reflects the conditions under which a conditional is false. The two weakening principles for the conditional that were noted in 5.3.2 directly reflect the two cases under which a conditional is true—when its consequent is true and when its antecedent is false.

$$\psi \Rightarrow \varphi \rightarrow \psi$$
$$\overline{\varphi} \Rightarrow \varphi \rightarrow \psi$$

However, while the rule CR implementing the law for the conditional as a premise is vital if our set of rules is sufficient, the rule that implements these weakening principles is optional like all attachment rules and is probably the least important of them.
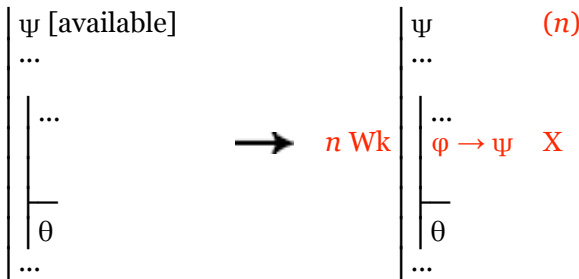

Fig. 5.4.2-1. Developing a derivation at stage *n* by adding an inactive conditional whose consequent is available.
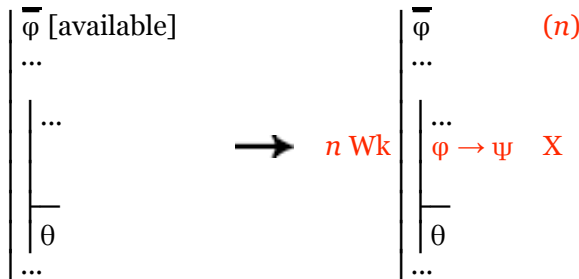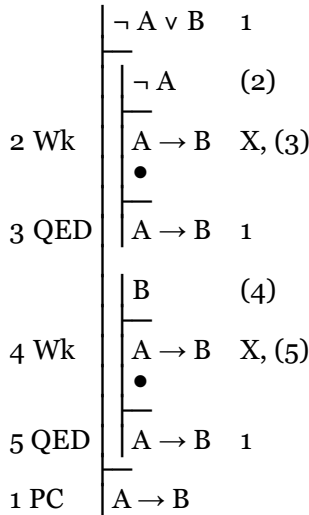

Fig. 5.4.2-2. Developing a derivation at stage *n* by adding an inactive conditional whose antecedent is barred by an available resource.

Much of the value of attachment rules lies in their use to assemble the auxiliary resource required to apply detachment rules. And, in natural arguments, the auxiliary resources of

detachment rules are less often conditionals than the other forms of sentence we can conclude by attachment rules. So we must look elsewhere for natural examples of the use of weakening for the conditional. As one example, consider the entailment ¬ A ∨ B ⇒ A → B. This can be established quickly by the use of CP and MTP, but if instead the disjunction is exploited to plan for a proof by cases, Wk for the conditional provides the most natural way to complete the case arguments.

$$
\begin{array}{lll}
 & \neg\,A \vee B & 1 \\[4pt]
 & \quad \neg\,A & (2) \\[2pt]
 2\ \text{Wk} & \quad A \to B \quad X,\,(3) \\
 & \quad \bullet \\[4pt]
 3\ \text{QED} & \quad A \to B \quad 1 \\[6pt]
 & \quad B & (4) \\[2pt]
 4\ \text{Wk} & \quad A \to B \quad X,\,(5) \\
 & \quad \bullet \\[4pt]
 5\ \text{QED} & \quad A \to B \quad 1 \\[6pt]
 1\ \text{PC} & A \to B
\end{array}
$$

A derivation showing that ¬ (A → B) ⇒ A ∧ ¬ B provides a similar example of the use of these rules.

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