5.4.1. Last resorts

The detachment rules for the conditional—and especially MPP—will be the ways of exploiting conditional resources that you will use the most. However, they cannot cover all cases because both require the presence of a second premise as an available resource. So we need a fully general way of taking account of conditional resources.

Since any open gap will eventually turn into a *reductio* argument, it is enough that we have a way of exploiting conditionals in such arguments. An entailment

$$\Gamma, \varphi \rightarrow \psi \Rightarrow \bot$$

says that $\varphi \rightarrow \psi$ is inconsistent with Γ, and it will hold if and only if φ is false in every possible world in which all members of Γ are true. But the conditional $\varphi \rightarrow \psi$ is false only when ψ is false while φ is true. So the displayed entailment says that in any world in which all members of Γ are true, we will find φ true and ψ false. But that tells us both that φ is entailed by Γ and that ψ is inconsistent with it. This way of describing the requirements for the validity a *reductio* with a conditional premise is our ***law for the conditional as a premise***:

$\Gamma, \varphi \rightarrow \psi \Rightarrow \bot$ if and only if both $\Gamma \Rightarrow \varphi$ and $\Gamma, \psi \Rightarrow \bot$.

That is, a conditional $\varphi \rightarrow \psi$ is excluded by a set Γ if and only if its antecedent φ is entailed by Γ and its consequent ψ is excluded by Γ.

In terms of the metaphor of inference tickets, the first law says that we can get to an absurd conclusion given Γ and the ticket $\varphi \rightarrow \psi$ if and only if Γ will get us to φ, the point of departure on our ticket, and then from its destination, ψ, on to the absurd conclusion. The "if" part of this holds also for conclusions that are not absurd, but the "only if" part does not. In particular, the fact that $\Gamma, \varphi \rightarrow \psi \Rightarrow \chi$ does not insure that $\Gamma \Rightarrow \varphi$ when χ is not absurd. We may be able to get to χ given Γ and the ticket $\varphi \rightarrow \psi$ without being able to get there via φ.

We will call the rule based on this principle, ***Rejecting a Conditional*** (RC). It is shown in Figure 5.4.1-2.

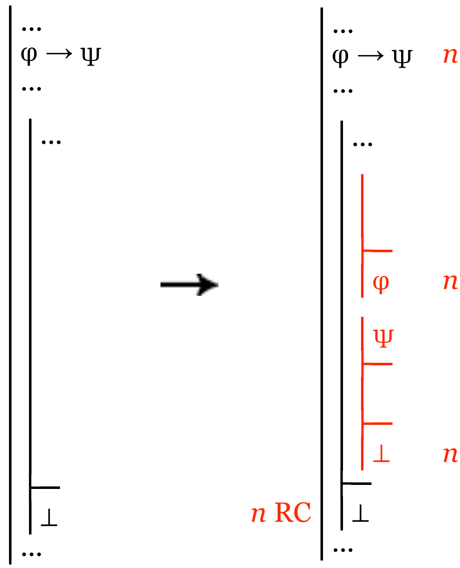
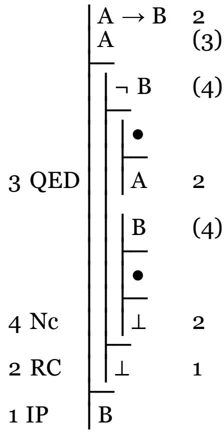


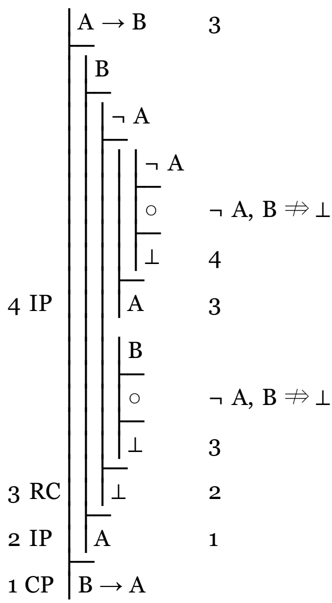
Fig. 5.4.1-2. Developing a *reductio* derivation at stage *n* by exploiting a conditional.

When we apply RC, we divide the gap into two, with the aim of showing that the antecedent of the conditional is entailed by our other resources and that its consequent is inconsistent with them. This is what is required to show that the conditional itself is inconsistent with our other resources, which is why we say that our aim is to ***reject*** the conditional. While this way of thinking about the rule is the most appropriate one given its place in the system of derivations, it can be thought of as a way of planning to use an inference ticket $\varphi \to \psi$ by planning to reach the point of departure φ and planning to get from the destination ψ to the goal $\bot$. From this point of view, we use the ticket to take us from the goal of the first of these open gaps to the assumption of the second.

Although MPP and MTT are more central to the deductive inference for the conditional than are MTP and MPT to inferences involving disjunction, negation, and conjunction, all detachment rules are dispensable. One role of RC is to exploit conditionals when detachment rules are not used, and one of the simplest example of its use is the following derivation which establishes the validity of *modus ponens* without use of MPP or MTT:



A more typical use of RC is a case we never have the second premise required in order to apply MPP or MTT, as in the following derivation, which shows that the conditional does not obey a commutative principle:



A	B	A → B	/	B → A
F	T	Ⓣ		Ⓕ

And, as is the case in this example, RC will serve us as a last resort for exploiting conditional resources before reaching a dead end in a derivation that fails.