5.3.xa. Exercise answers

1. a.

$$
\begin{array}{lll}
 & B \rightarrow C & 3 \\
 & A \rightarrow B & 2 \\
\hline
 & \quad A & (2) \\
\text{2 MPP} & \quad B & (3) \\
\text{3 MPP} & \quad C & (4) \\
 & \quad \bullet & \\
\hline
\text{4 QED} & \quad C & 1 \\
\hline
\text{1 PC} & A \rightarrow C &
\end{array}
$$

b.

$$
\begin{array}{lll}
 & A \rightarrow B & 2 \\
 & C \leftarrow B & 3 \\
 & C \rightarrow D & 4 \\
\hline
 & \quad A & (2) \\
\text{2 MPP} & \quad B & (3) \\
\text{3 MPP} & \quad C & (4) \\
\text{4 MPP} & \quad D & (5) \\
 & \quad \bullet & \\
\hline
\text{5 QED} & \quad D & 1 \\
\hline
\text{1 CP} & A \rightarrow D &
\end{array}
$$

c.

$$
\begin{array}{lll}
 & A \rightarrow (B \rightarrow C) & 3 \\
\hline
 & \quad A \rightarrow B & 4 \\
\hline
 & \quad\quad A & (3),(4) \\
\text{3 MPP} & \quad\quad B \rightarrow C & 5 \\
\text{4 MPP} & \quad\quad B & (5) \\
\text{5 MPP} & \quad\quad C & (6) \\
 & \quad\quad \bullet & \\
\hline
\text{6 QED} & \quad\quad C & 2 \\
\hline
\text{2 CP} & \quad A \rightarrow C & 1 \\
\hline
\text{1 CP} & (A \rightarrow B) \rightarrow (A \rightarrow C) &
\end{array}
$$

d.

	$A \rightarrow (B \rightarrow C)$	3
	$A \rightarrow \neg C$	4
	B	(5)
	A	(3),(4)
3 MPP	$B \rightarrow C$	5
4 MPP	$\neg C$	(6)
5 MPP	C	(6)
	$\bullet$	
6 Nc	$\bot$	2
2 RAA	$\neg A$	1
1 CP	$B \rightarrow \neg A$	

e.

	$\neg A$	(2)
	A	
	$\bullet$	
2 QED	$\neg A$	1
1 CP	$A \rightarrow \neg A$	

	$A \rightarrow \neg A$	2
	A	(2),(3)
2 MPP	$\neg A$	(3)
	$\bullet$	
3 Nc	$\bot$	1
1 CP	$\neg A$	

f.

	$A \rightarrow B$	2
	$\neg B$	(2)
2 MTT	$\neg A$	(3)
	$\bullet$	
3 QED	$\neg A$	1
1 CP	$\neg B \rightarrow \neg A$	

	$\neg B \rightarrow \neg A$	2
	A	(2)
2 MTT	B	(3)
	$\bullet$	
3 QED	B	1
1 CP	$A \rightarrow B$	

g.

	$A \rightarrow B$	3
	$A \wedge \neg B$	2
2 Ext	A	(3)
2 Ext	$\neg B$	(4)
3 MPP	B	(4)
	$\bullet$	
4 Nc	$\bot$	1
1 RAA	$\neg (A \wedge \neg B)$	

	$\neg (A \wedge \neg B)$	2
	A	(2)
2 MPT	B	(3)
	$\bullet$	
3 QED	B	1
1 CP	$A \rightarrow B$	

h.

	A → (B → C)	3
	A ∧ B	2
2 Ext	A	(3)
2 Ext	B	(4)
3 MPP	B → C	4
4 MPP	C	(5)
	•	
5 QED	C	1
1 CP	(A ∧ B) → C	

	(A ∧ B) → C	4
	A	(5)
	B	(6)
	¬ C	(4)
4 MTT	¬ (A ∧ B)	5
5 MPT	¬ B	(6)
	•	
6 Nc	⊥	3
3 IP	C	2
2 CP	B → C	1
1 CP	A → (B → C)	

i.

	(A → B) ∧ (A → C)	1
1 Ext	A → B	3
1 Ext	A → C	4
	A	(3),(4)
3 MPP	B	(6)
4 MPP	C	(7)
	•	
6 QED	B	5
	•	
7 QED	C	5
5 Cnj	B ∧ C	2
2 CP	A → (B ∧ C)	

	A → (B ∧ C)	3,7
	A	(3)
3 MPP	B ∧ C	4
4 Ext	B	(5)
4 Ext	C	
	•	
5 QED	B	2
2 CP	A → B	1
	A	
7 MPP	B ∧ C	8
8 Ext	B	
8 Ext	C	(9)
	•	
9 QED	C	6
6 QED	A → C	1
1 Cnj	(A → B) ∧ (A → C)	

j. Stages 3-5 and 7-11 in the derivation at the right could have taken analogous forms; they are varied here to show two approaches, one using attachment rules and the other without them.

	(A → C) ∧ (B → C)	1			(A v B) → C		4,8
1 Ext	A → C	4				A	(3)
1 Ext	B → C	6		3 Wk		A v B	X,(4)
		A v B	3		4 MPP		C	(5)
			A	(4)	5 QED		C	2
4 MPP		C	(5)	2 CP	A → C		1
5 QED		C	3			B	(11)
		B	(6)			¬ C	(8)
6 MPP		C	(7)	8 MTT		¬ (A v B)	9
7 QED		C	3				¬ A	•
3 PC	C	2		11 QED			B	10
2 CP	(A v B) → C			10 PE		A v B	9
				9 CR		⊥	7
				7 IP		C	6
				6 CP	B → C		1
				1 Cnj	(A → C) ∧ (B → C)		

k. Parallel arguments are again completed differently in the two gaps of each derivation—in the first, to show approaches with attachment rules and without them and, in the second, to show two ways of using attachment rules.

	(A → B) ∧ (B → C)	1			(A ∨ B) → (B ∧ C)	4,10
1 Ext	A → B	4			A	(3)
1 Ext	B → C	5,10	3 Wk		A ∨ B	X,(4)
	A ∨ B	3	4 MPP		B ∧ C	5
	A	(4)	5 Ext		B	(6)
4 MPP	B	(5)	5 Ext		C	
5 MPP	C				•	
6 Adj	B ∧ C	X,(7)	6 QED		B	2
	•		2 CP		A → B	1
7 QED	B ∧ C	3			B	(11)
	B	(9),(10)			¬ C	(9)
	•				¬ (B ∧ C)	(10)
9 QED	B	8	9 Wk		¬ (A ∨ B)	(12)
10 MPP	C	(11)	10 MTT		A ∨ B	(12)
	•		11 Wk		•	
11 QED	C	8	12 Nc		⊥	8
8 Cnj	B ∧ C	3	8 IP		C	7
3 PC	B ∧ C	2	7 CP		B → C	1
2 CP	(A ∨ B) → (B ∧ C)		1 Cnj		(A → B) ∧ (B → C)	

2. **d.**   *If Ann was there, then Carol was there if Bill was*
Carol wasn't there if Ann was
Ann wasn't there if Bill was

f.   *If Ann was there, Bill was, too*
If Bill wasn't there, Ann wasn't either

g.   *If Ann was there, Bill was there*
Ann wasn't there without Bill being there

k.   *If Ann was there, Bill was there; and if Bill was there, Carol was there*
If either Ann or Bill was there, then both Bill and Carol were there