5.3.s. Summary

The truth conditions of the conditional recall the definition of implication. Indeed, an implication $\varphi \Rightarrow \psi$ will hold if and only if the conditional $\varphi \to \psi$ is a tautology. We can apply similar ideas to conditionals that are conclusions from factual premises by considering a notion of relative implication, implication depending on factual information. This idea appears in our law for the conditional as a conclusion. An entailment $\Gamma \Rightarrow \varphi \to \psi$ holds when $\Gamma, \varphi \Rightarrow \psi$—i.e., when ψ is implied by φ given the further premises Γ. The first of these entailments is a conditionalization of the second, and the second asserts the validity of a hypothetical argument. So an argument with a conditional conclusion is valid if and only if the hypothetical argument it conditionalizes is also valid. The derivation rule implementing this idea is Conditional Proof (CP).

The detachment principles for the conditional include the well-known *modus ponendo ponens* (usually called *modus ponens*), which is implemented as a rule Modus Ponendo Ponens (MPP), and a second detachment principle *modus tollendo tollens* (usually called *modus tollens*), which is implemented as a rule Modus Tollendo Tollens (MTT). *Modus ponens* in particular can be understood as the use of a conditional as an inference ticket licensing transitions from its antecedent to its consequent.

Glen Helman 25 Aug 2005