5.2.2. Necessary and sufficient conditions

Like *if*-conditionals, *only-if*-conditionals in the indicative voice carry implicatures, but their implicatures are different. This difference can be captured by the phrases ***necessary condition*** and ***sufficient condition***. Consider the following sentences:

The match burned only if oxygen was present.
The match burned if it was struck.

Each carries, as an implicature, the suggestion of a connection between the burning of the match and some other state or event. In the first, the suggestion is that the presence of oxygen was required for the match to burn, that it was a necessary condition without which combustion could not occur. The suggestion of the second is that the striking of the match would have been enough for it to burn, that it would have been a sufficient condition. These necessary and sufficient conditions might be described as ***causal***; they concern states whose absence can prevent an event from occurring or other events which are enough to bring it about.

Another kind of necessary and sufficient conditions could be described as ***epistemic*** since they concern grounds for reasonable belief. For example, we might say this.

If the match burned, oxygen was present.

In making this assertion, we do not mean to suggest that the burning of the match would have brought about the presence of oxygen but rather that the burning would be evidence of oxygen's presence. Combustion would give us sufficient grounds for believing that oxygen was present, so it is epistemically sufficient. On the other hand, we might say this:

The switch was thrown only if the light was on.

Suppose it is known that the switch is in different room from the light. The sentence would not suggest that the light was required for the switch to be thrown but rather that the light being on served as a test of the belief that the switch was thrown. That is, seeing that the light was not on would lead us to reject a belief that the switch was thrown. Epistemic conditions of both sorts are sometimes referred to as *signs* or *marks*.

Now, statements of necessary and sufficient conditions can themselves be understood as connectives, ones that we might express more explicitly in the following way:

The truth of φ *is a necessary condition for the truth of* ψ
The truth of φ *is a sufficient condition for the truth of* ψ.

A compound of either of these forms is plainly not truth-functional. Knowing, for example, that φ and ψ are both true will not tell us whether either is a necessary or a sufficient condition for the other. So necessary and sufficient conditions are not strictly within our purview. But, since they attach to indicative *if*- and *only-if*-conditionals as implicatures, we need to be aware of them because they can make certain ways of restating such conditionals more natural than others.

When checking that the form ¬ ψ ← ¬ φ has that same truth table as ψ *only if* φ, you may have noticed that the simpler form ψ → φ also has the same table. This might suggest that as a first step in analyzing ψ *only if* φ we could rephrase it as *If* ψ *then* φ. However, to do so would often wreak such havoc on the implicatures that the paraphrase would sound crazy. In saying ψ *only if* φ, we suggest that the truth of φ is a necessary condition for the truth of ψ while in saying *If* ψ *then* φ, we suggest that the truth of ψ is a sufficient condition for the truth of φ. And sufficiency and necessity are not simple converse relations like *parent of* and *child of*.

As we saw in the examples above, a causally sufficient condition for an event may have the event as an epistemically necessary condition, and a causally necessary condition may have the event as an epistemically sufficient condition. However, in making such shifts we are changing the meaning of a sentence in a noticeable way. This problem becomes especially severe in the case of conditionals concerning the future, where causal and epistemic conditions tend to coincide. A meteorologist would certainly not be prepared to use the following interchangeably:

It will rain tomorrow only if the front moves through.
If it rains tomorrow, the front will move through.

We could do a bit better in this case by adjusting tenses to get *If it rains tomorrow, then front will have moved through*, but we still have shifted from causal to epistemic implicatures.

The analysis *only-if*-conditionals that we do employ amounts to a paraphrase of ψ *only if* φ by *It's not the case that* ψ *if it's not the* *case that* φ. And this paraphrase tends to avoid such problems with implicatures. But it only *tends* to avoid them because our description of the implicatures of *if-* and *only-if*-conditionals in terms of necessary and sufficient conditions is still an oversimplified account of the relation between them. For example, I might express my conviction that the temperature is high by using the sentence *It's under 80° only if it's over 75°*. Here the paraphrase *If it's under 80°, it's over 75°* works well even though it is the sort of paraphrase that failed in earlier examples; and a paraphrase of the sort we used in those examples—namely, *If it isn't over 75°, then it isn't under 80°*—sounds crazy.

Now its being over 75° could be a necessary condition for its being under 80° only if we take it for granted that it is hot. And the point of the initial sentence is more to commit the speaker to this presumption than to suggest the existence of a necessary condition. The sentence *If it isn't over 75°, then it isn't under 80°* cannot play this role since it pointedly leaves open just the sort of case whose failure the original sentence is designed to suggest.

This sort of example shows that the implicatures of *if*- and *only-if*-conditionals can be sufficiently independent that the latter cannot be expressed in terms of the former. However, if we paraphrase using negation (rather than reversing main and subordinate clauses), the difference in implicatures will usually not be too great. The moral for our purposes is then that a paraphrase of ψ *only if* φ by ¬ ψ *if* ¬ φ will usually not be too jarring though *if* ψ *then* φ may be better in a few cases.

There is a final complication in dealing with *if* and *only if* that it is also a result of their implicatures. Conditionals of the two sorts can often be difficult to distinguish because a conditional of one sort carries a conditional of the other sort as an implicature. For example, imagine I were speaking of a farm in a year when corn yields have been affected by drought. If I were to assert the sentence

They will make a profit only if they get over $2.50 a bushel,

I would be understood to believe not only that this price was necessary for a profit but also that it was sufficient, and it seems that I would agree with the following:

They will make a profit if they get over $2.50 a bushel.

But this is only an implicature and, unlike the suggestion that the price is a necessary condition for making a profit, the suggestion that it is also sufficient is one that is easily canceled. If I wanted to avoid the implicature, I might have used the sentence

They will make a profit only if they get over $2.50 a bushel, and even that might not be enough

and I would not have contradicted myself by saying this.

Moreover, the implicature of an *if*-conditional by an *only-if*-conditional, or vice versa, does not always arise. We would usually take the forecast *It will rain tomorrow only if the front moves through* to suggest that the passing of the front would produce rain; but during a severe drought, when rain seems very unlikely, a forecaster might not need to add the canceling clause *and it might stay dry even if the front does move through*. So, while implicatures may conceal the difference between them, ψ *if* φ and ψ *only if* φ really are different in content from each other.

This means that the assertion of both conditionals, as in the form ψ *if and only if* φ, is not redundant. This sort of compound is known as the ***biconditional***. Its analysis would lead us to the form

$$(\psi \leftarrow \varphi) \land (\neg\, \psi \leftarrow \neg\, \varphi)$$

or, with rightwards arrows,

$$(\varphi \rightarrow \psi) \land (\neg\, \varphi \rightarrow \neg\, \psi)$$

Biconditionals appear often in definitions, and calculating the truth table for this form will show why. A biconditional is true when the components φ and ψ are both true and also when they are both false, so this form enables us to say that two sentences have the same truth value without saying what that value is.