

5.1.1. Conditions

The use of *or* is not the only way of hedging what we say. Instead of hedging a claim by offering an alternative, we can limit what we rule out to a certain range of possibilities. For example, instead of saying *It will rain tomorrow*, a forecaster might say *It will rain tomorrow if the front moves through*. The subordinate clause *if the front moves through* limits the forecaster's commitment to rain tomorrow to cases where the front does move through. If it does not move through, the forecaster's prediction cannot be faulted even if it does not rain.

We will refer to the connective marked by *if* as the **(if-)conditional** and to sentences of the form ψ *if* ϕ as **(if-)conditionals**. The qualification *if-* is used here to distinguish this connective from connectives associated with *only if* and *unless* that we will consider in [5.2](#). The three connectives are closely related, we will refer to all three as **conditionals**. However, the *if*-conditional is the most important of the three we will consider, and a reference to “the conditional” without qualification will be to it. Outside of contexts where we are discussing several sorts of conditional sentence, a reference to “conditionals” will be to the various compounds formed using it rather than to the three sorts of connective. In fact, we will analyze the other two connectives in a way that makes the *if*-conditional the main component of the result, so compounds formed using the other two connectives will count as special sorts of *if*-conditionals.

Although we take the word *if*, like the words *and* and *or*, to mark a two-place connective, it raised somewhat different grammatical issues. Since it is used mainly to join full clauses, there is less often a need to fill out the expressions it joins to get full sentences (though, of course, pronominal reference from one component to another must still be removed). And there are special problems associated with it. The conditional is an asymmetric connective: it makes a difference which component is having its content trimmed and which expresses the condition used to trim it. For example, there is a considerable difference between the following sentences:

Mike entered the contest if he won the prize
Mike won the prize if he entered the contest.

The first is a truism about contests and merely rules out cases of

Mike winning the prize without entering the contest. On the other hand, the second suggests confidence in Mike's success and rules out cases where he entered the contest without winning.

Still, no fixed order between the two clauses of a conditional is imposed by English syntax. Like other subordinate clauses, *if*-clauses can be moved to the beginning of the sentence. Thus the two sentences above could be rephrased, respectively, as the following:

If Mike won the prize, he entered the contest
If Mike entered the contest, he won the prize

Sometimes the word *then* will precede the main clause in such cases; but, as the examples above show, this is not necessary.

We will use the asymmetric notation \rightarrow (the **rightwards arrow**) or \leftarrow (the **leftwards arrow**) for the conditional. The subordinate *if*-clause will contribute the component at the tail of the arrow, and the main clause of a conditional sentence will contribute the component at the head. We will refer to these two components, respectively, as the **antecedent** (i.e., what comes before, in the direction of the arrow) and the **consequent** (what comes after). Since the difference between the conditioned claim and what it is conditional on is marked by the difference between the two ends of the arrow, the order in which we write these components makes no difference provided that the arrow points from the antecedent to the consequent. For example, *Adam opened the package if it had his name on it* could be written as either of the following:

Adam opened the package \leftarrow *the package had Adam's name on it*

The package had Adam's name on it \rightarrow *Adam opened the package*

This means that the reordering of clauses in English can be matched by our symbolic notation, with $\phi \rightarrow \psi$ corresponding to *If* ϕ *then* ψ and $\psi \leftarrow \phi$ corresponding to ψ *if* ϕ . When we are not attempting to match the word order of English sentence, the rightwards arrow will be the preferred notation, and generalizations about conditionals will usually be stated only for the form $\phi \rightarrow \psi$.

We will use **if** ϕ **then** ψ as English notation for $\phi \rightarrow \psi$. Here the word **if** plays the role of a left parenthesis (as **both** and

either do). We will not often use English notation for the leftwards arrow, but it can help in understanding the relation of the two to have some available. If we are to have anything corresponding to the form $\psi \leftarrow \phi$, we will put *if* between the two components, so we need another word to the role of left parenthesis. English usage provides no natural choices, so we will have to be a bit arbitrary. The interjection *yes* does not disturb the grammar of the surrounding sentence, so it can be easily placed where we want it. So we will write *yes* ψ *if* ϕ as our English notation for the form $\psi \leftarrow \phi$. This way of tying the words *yes* and *if* is not backed up by an intuitive understanding of English, so the *yes* in the form *yes* ψ *if* ϕ does not help in understanding the symbolic form. On the other hand, it does not interfere with the help that *if* provides; and, as an interjection, it can help to mark breaks in a sentence in the way punctuation does.

The leftwards arrow \leftarrow is the easier of the two to accommodate if we look for a simple English substitute to use along with parentheses, for it corresponds directly to *if*. We will not often need to use English notation with parentheses in the case of conditionals, so finding something for the rightwards arrow \rightarrow is not a pressing practical problem. However, the way this problem is typically solved emphasizes an important point about the conditional

Of course, we cannot use *if* also for the rightwards arrow. And, even if we were not using *if* for the leftwards arrow, it would not work for \rightarrow since *if* in English must precede rather than follow the subordinate clause. And *then* will not do either since *if* (unlike *both* or *either*) bears the meaning of the connective in English. The usual approach is to look further afield and employ the word *implies*. Lacking a better alternative, we will follow this practice and use the word *implies* (in this typeface) as an English version of \rightarrow to use with parentheses.

There is some danger of confusion in doing this, for we have used *implies* as a synonym for *entails* in the case of a single premises, and the arrows \rightarrow and \Rightarrow have quite different meanings. In particular, the notation $\phi \rightarrow \psi$ refers to a sentence that speaks only of the actual world while, in saying that $\phi \Rightarrow \psi$, we make a claim about all possible worlds. One way to avoid the confusion is to say that $\phi \rightarrow \psi$ expresses **material implication** while, when

saying that $\phi \Rightarrow \psi$, we express **logical implication**. We will discuss this distinction further in [5.3.1](#); but, for now, we can note that this terminology is intended to capture a distinction between a claim about what is a matter of fact on the one hand and a claim about logical necessity on the other. And, however we describe the difference, this is a case where the typeface definitely matters, for

ϕ **implies** ψ

is the use of an English word to provide an alternative notation for $\phi \rightarrow \psi$ while

ϕ implies ψ

is our way, in ordinary English, of saying what is expressed in notation as $\phi \Rightarrow \psi$.

To give an example of some of this notation in action, let us return to the idea that a conditional serves to trim the content of its consequent. This can be expressed in symbolic notation as the entailment

$\psi \Rightarrow \phi \rightarrow \psi$

which says that the argument $\psi / \phi \rightarrow \psi$ is a valid one. If we use English notation for the conditional, we might express the same entailment as either

$\psi \Rightarrow$ **if** ϕ **then** ψ

or

$\psi \Rightarrow \phi$ **implies** ψ

and we express the relation in English, using *implies* to express entailment, by saying that ψ implies $\phi \rightarrow \psi$, that ψ implies **if** ϕ **then** ψ , or that ψ implies ϕ **implies** ψ . Of course, because we have all these options, we have many ways of avoiding potentially confusing expressions; but trying to discern the meaning of a potentially confusing but really unambiguous expression is a good exercise in sorting out the range of concepts we are working with.