

4.3.xa. Exercise answers

1. a.

	$A \wedge B$	1
	A	(2)
1 Ext	B	
1 Ext	$A \vee B$	X,(3)
2 Wk	•	
	$A \vee B$	
3 QED		

b.

	$A \wedge B$	1
	A	
1 Ext	B	(2)
1 Ext	$B \vee C$	X,(3)
2 Wk	•	
	$B \vee C$	
3 QED		

c.

	$A \vee B$	1
	$\neg A$	(1)
	B	(2)
1 MTP	•	
	B	
2 QED		

d. Although the following is a possible approach, the derivation in 4.2.xa is probably more natural:

	$A \vee (A \wedge B)$	2
	$\neg A$	(2),(4)
	$A \wedge B$	3
2 MTP	A	(4)
3 Ext	B	
3 Ext	•	
	\perp	1
4 Nc		
	A	
1 IP		

e.

	$A \vee B$	3
	$\neg (A \wedge C)$	2
	$\neg (B \wedge C)$	4
	C	(2),(5)
	$\neg A$	(3)
2 MPT	B	(4)
3 MTP	$\neg C$	(5)
4 MPT	•	
	\perp	1
5 Nc		
	$\neg C$	
1 RAA		

<p>c.</p> <table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-right: 1px solid black; padding: 5px;">$(A \vee B) \vee C$</td><td style="padding: 5px;">3</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$\neg A$</td><td style="padding: 5px;">(4)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$\neg C$</td><td style="padding: 5px;">(3)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$A \vee B$</td><td style="padding: 5px;">4</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">B</td><td style="padding: 5px;">(5)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">•</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">B</td><td style="padding: 5px;">2</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$B \vee C$</td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$A \vee (B \vee C)$</td><td></td></tr> </table> <p>3 MTP 4 MTP 5 QED 2 PE 1 PE</p>	$(A \vee B) \vee C$	3	$\neg A$	(4)	$\neg C$	(3)	$A \vee B$	4	B	(5)	•		B	2	$B \vee C$	1	$A \vee (B \vee C)$		<table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-right: 1px solid black; padding: 5px;">$A \vee (B \vee C)$</td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">A</td><td style="padding: 5px;">(2)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$A \vee B$</td><td style="padding: 5px;">X,(3)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$(A \vee B) \vee C$</td><td style="padding: 5px;">X,(4)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">•</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$(A \vee B) \vee C$</td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$B \vee C$</td><td style="padding: 5px;">5</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">B</td><td style="padding: 5px;">(6)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$A \vee B$</td><td style="padding: 5px;">X,(7)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$(A \vee B) \vee C$</td><td style="padding: 5px;">X,(8)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">•</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$(A \vee B) \vee C$</td><td style="padding: 5px;">5</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">C</td><td style="padding: 5px;">(9)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$(A \vee B) \vee C$</td><td style="padding: 5px;">(10)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">•</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$(A \vee B) \vee C$</td><td style="padding: 5px;">5</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$(A \vee B) \vee C$</td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$(A \vee B) \vee C$</td><td></td></tr> </table> <p>2 Wk 3 Wk 4 QED 6 Wk 7 Wk 8 QED 9 Wk 10 QED 5 PC 1 PC</p>	$A \vee (B \vee C)$	1	A	(2)	$A \vee B$	X,(3)	$(A \vee B) \vee C$	X,(4)	•		$(A \vee B) \vee C$	1	$B \vee C$	5	B	(6)	$A \vee B$	X,(7)	$(A \vee B) \vee C$	X,(8)	•		$(A \vee B) \vee C$	5	C	(9)	$(A \vee B) \vee C$	(10)	•		$(A \vee B) \vee C$	5	$(A \vee B) \vee C$	1	$(A \vee B) \vee C$	
$(A \vee B) \vee C$	3																																																						
$\neg A$	(4)																																																						
$\neg C$	(3)																																																						
$A \vee B$	4																																																						
B	(5)																																																						
•																																																							
B	2																																																						
$B \vee C$	1																																																						
$A \vee (B \vee C)$																																																							
$A \vee (B \vee C)$	1																																																						
A	(2)																																																						
$A \vee B$	X,(3)																																																						
$(A \vee B) \vee C$	X,(4)																																																						
•																																																							
$(A \vee B) \vee C$	1																																																						
$B \vee C$	5																																																						
B	(6)																																																						
$A \vee B$	X,(7)																																																						
$(A \vee B) \vee C$	X,(8)																																																						
•																																																							
$(A \vee B) \vee C$	5																																																						
C	(9)																																																						
$(A \vee B) \vee C$	(10)																																																						
•																																																							
$(A \vee B) \vee C$	5																																																						
$(A \vee B) \vee C$	1																																																						
$(A \vee B) \vee C$																																																							

The derivation at the right can be compared to the one in [4.2.3](#)

<p>d.</p> <table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-right: 1px solid black; padding: 5px;">$A \vee (B \wedge \neg B)$</td><td style="padding: 5px;">2</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$\neg A$</td><td style="padding: 5px;">(2)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$B \wedge \neg B$</td><td style="padding: 5px;">3</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">B</td><td style="padding: 5px;">(4)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$\neg B$</td><td style="padding: 5px;">(4)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">•</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">\perp</td><td style="padding: 5px;">4</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">A</td><td></td></tr> </table> <p>2 MTP 3 Ext 3 Ext 4 Nc 1 IP</p>	$A \vee (B \wedge \neg B)$	2	$\neg A$	(2)	$B \wedge \neg B$	3	B	(4)	$\neg B$	(4)	•		\perp	4	A		<table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-right: 1px solid black; padding: 5px;">A</td><td style="padding: 5px;">(1)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$A \vee (B \wedge \neg B)$</td><td style="padding: 5px;">X,(2)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">•</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$A \vee (B \wedge \neg B)$</td><td></td></tr> </table> <p>1 Wk 2 QED</p>	A	(1)	$A \vee (B \wedge \neg B)$	X,(2)	•		$A \vee (B \wedge \neg B)$	
$A \vee (B \wedge \neg B)$	2																								
$\neg A$	(2)																								
$B \wedge \neg B$	3																								
B	(4)																								
$\neg B$	(4)																								
•																									
\perp	4																								
A																									
A	(1)																								
$A \vee (B \wedge \neg B)$	X,(2)																								
•																									
$A \vee (B \wedge \neg B)$																									

<p>e.</p> <table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-right: 1px solid black; padding: 5px;">$\neg (A \vee B)$</td><td style="padding: 5px;">(4),(7)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">A</td><td style="padding: 5px;">(3)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$A \vee B$</td><td style="padding: 5px;">X,(4)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">•</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">\perp</td><td style="padding: 5px;">2</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$\neg A$</td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">B</td><td style="padding: 5px;">(6)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$A \vee B$</td><td style="padding: 5px;">X,(7)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">•</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">\perp</td><td style="padding: 5px;">5</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$\neg B$</td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$\neg A \wedge \neg B$</td><td></td></tr> </table> <p>3 Wk 4 Nc 2 RAA 6 Wk 7 Nc 5 RAA 1 Cnj</p>	$\neg (A \vee B)$	(4),(7)	A	(3)	$A \vee B$	X,(4)	•		\perp	2	$\neg A$	1	B	(6)	$A \vee B$	X,(7)	•		\perp	5	$\neg B$	1	$\neg A \wedge \neg B$		<table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-right: 1px solid black; padding: 5px;">$\neg A \wedge \neg B$</td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$\neg A$</td><td style="padding: 5px;">(3)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$\neg B$</td><td style="padding: 5px;">(4)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$A \vee B$</td><td style="padding: 5px;">3</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">B</td><td style="padding: 5px;">(4)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">•</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">\perp</td><td style="padding: 5px;">2</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">$\neg (A \vee B)$</td><td></td></tr> </table> <p>1 Ext 1 Ext 3 MTP 4 Nc 2 RAA</p>	$\neg A \wedge \neg B$	1	$\neg A$	(3)	$\neg B$	(4)	$A \vee B$	3	B	(4)	•		\perp	2	$\neg (A \vee B)$	
$\neg (A \vee B)$	(4),(7)																																								
A	(3)																																								
$A \vee B$	X,(4)																																								
•																																									
\perp	2																																								
$\neg A$	1																																								
B	(6)																																								
$A \vee B$	X,(7)																																								
•																																									
\perp	5																																								
$\neg B$	1																																								
$\neg A \wedge \neg B$																																									
$\neg A \wedge \neg B$	1																																								
$\neg A$	(3)																																								
$\neg B$	(4)																																								
$A \vee B$	3																																								
B	(4)																																								
•																																									
\perp	2																																								
$\neg (A \vee B)$																																									

f.	$\frac{\frac{\frac{\frac{\neg(A \wedge B)}{2}}{A \quad (2)}{2 \text{ MPT}}{\neg B \quad (3)}{\bullet}}{\neg B \quad 1} \quad 3 \text{ QED}}{\neg A \vee \neg B} \quad 1 \text{ PE}$		$\frac{\frac{\frac{\frac{\neg A \vee \neg B}{3}}{A \wedge B \quad 2}}{A \quad (3)}{2 \text{ Ext}}{B \quad (4)}{3 \text{ MTP}}{\neg B \quad (4)}{\bullet}}{\perp \quad 1} \quad 4 \text{ Nc}$ $\frac{}{\neg(A \wedge B)} \quad 1 \text{ RAA}$
-----------	---	--	--

3. a. This derivation is unchanged from 4.2.xa

	$\frac{\frac{\frac{\frac{\frac{A \vee B}{2}}{A}}{B}}{\frac{A}{\circ} \quad A, B \not\Rightarrow \perp}}{\perp \quad 2}}{\frac{B}{\circ} \quad A, B \not\Rightarrow \perp}}{\perp \quad 2}}{\perp \quad 1} \quad 2 \text{ PC}$ $\frac{}{\neg B} \quad 1 \text{ RAA}$		<table style="border-collapse: collapse; margin-left: auto; margin-right: auto;"> <tr> <td style="border-right: 1px solid black; padding: 5px;">$A \vee B$</td> <td style="padding: 5px;">$A \vee B, A / \neg B$</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">T T</td> <td style="padding: 5px;">(T) (T) (F)</td> </tr> </table>	$A \vee B$	$A \vee B, A / \neg B$	T T	(T) (T) (F)
$A \vee B$	$A \vee B, A / \neg B$						
T T	(T) (T) (F)						

b.	$\frac{\frac{\frac{\frac{\frac{A \vee (B \wedge C)}{3,8}}{\neg A \quad (3)}{3 \text{ MTP}}{B \wedge C \quad 4} \quad 4 \text{ Ext}}{B \quad (5)} \quad 4 \text{ Ext}}{\bullet}}{B \quad 2} \quad 5 \text{ QED}}{A \vee B \quad 1} \quad 2 \text{ PE}$ $\frac{\frac{\frac{\frac{\frac{\neg C \quad (7)}{\neg(B \wedge C) \quad X,(8)}{A \quad (8)}{\circ} \quad A, \neg C \not\Rightarrow \perp}}{\perp \quad 9}}{C \quad 1} \quad 6 \text{ IP}}{(A \vee B) \wedge C} \quad 1 \text{ Cnj}$		$\frac{\frac{\frac{\frac{(A \vee B) \wedge C \quad 1}{A \vee B \quad 3} \quad 1 \text{ Ext}}{C \quad (4)} \quad 1 \text{ Ext}}{\neg A \quad (3)}}{\frac{B \quad (4)}{B \wedge C \quad X,(5)} \quad 3 \text{ MTP}}{\bullet}}{B \wedge C \quad 2} \quad 4 \text{ Adj} \quad 5 \text{ QED}$ $\frac{}{A \vee (B \wedge C)} \quad 2 \text{ PE}$									
			<p>Each of the following divides the one open gap:</p> <table style="border-collapse: collapse; margin-left: auto; margin-right: auto;"> <tr> <td style="border-right: 1px solid black; padding: 5px;">$A \ B \ C$</td> <td style="border-right: 1px solid black; padding: 5px;">$A \vee (B \wedge C) / (A \vee B) \wedge C$</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">T T F</td> <td style="border-right: 1px solid black; padding: 5px;">(T) F T (F)</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">T F F</td> <td style="border-right: 1px solid black; padding: 5px;">(T) F T (F)</td> <td style="padding: 5px;"></td> </tr> </table>	$A \ B \ C$	$A \vee (B \wedge C) / (A \vee B) \wedge C$		T T F	(T) F T (F)		T F F	(T) F T (F)	
$A \ B \ C$	$A \vee (B \wedge C) / (A \vee B) \wedge C$											
T T F	(T) F T (F)											
T F F	(T) F T (F)											

Although the use of Wk and MTP shortens the whole first derivation, it actually delays the dead end, which would have been

reached after stage 7 if the first premise had been exploited by PC in the second gap. As in 4.2.xa, the second derivation is unnecessary once a dead-end gap is found in the first.

c.

	$\neg(A \vee B)$	(4)
	A	(3)
	B	
3 Wk	A \vee B	X,(4)
	●	
4 Nc	\perp	2
2 RAA	$\neg B$	1
1 PE	$\neg A \vee \neg B$	

	$\neg A \vee \neg B$	2
	A \vee B	3,4
	$\neg A$	(3)
3 MTP	B	
	○	$\neg A, B \Rightarrow \perp$
	\perp	2
	$\neg B$	(4)
4 MTP	A	
	○	A, $\neg B \Rightarrow \perp$
	\perp	2
2 PC	\perp	1
1 RAA	$\neg(A \vee B)$	

The following divide the first and second open gap, respectively:

A	B	$\neg A \vee \neg B$	$\neg(A \vee B)$
F	T	⊖ F	⊖ T
T	F	⊖ T	⊖ T