

4.2.xa. Exercise answers

1. a.

	$A \wedge B$	1	
1 Ext	A		
1 Ext	B		(3)
	$\neg A$		
	•		
3 QED	B		2
2 PE	$A \vee B$		

b.

	$A \wedge B$	1	
1 Ext	A		
1 Ext	B		(3)
	$\neg C$		
	•		
3 QED	B		2
2 PE	$B \vee C$		

c.

	$A \vee B$	1	
	$\neg A$		(3)
	A		(3)
	$\neg B$		
	•		
3 Nc	\perp		2
2 IP	B		1
	B		(4)
	•		
4 QED	B		1
1 PC	B		

d.

	$A \vee (A \wedge B)$	1
	A	(2)
	•	
2 QED	A	1
	$A \wedge B$	3
3 Ext	A	(4)
3 Ext	B	
	•	
4 QED	A	1
1 PC	A	

e.

	$A \vee B$	2
	$\neg (A \wedge C)$	3
	$\neg (B \wedge C)$	7
	C	(6),(10)
	A	(5)
	•	
5 QED	A	4
	•	
6 QED	C	4
4 Cnj	$A \wedge C$	3
3 CR	\perp	2
	B	(9)
	•	
9 QED	B	8
	•	
10 QED	C	8
8 Cnj	$B \wedge C$	7
7 CR	\perp	2
2 PC	\perp	1
1 RAA	$\neg C$	

f.	$A \wedge (B \vee C)$	1
1 Ext	A	(5)
1 Ext	$B \vee C$	2
	B	(6)
	<div style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;">$\neg C$</div>	
	<div style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;">•</div>	
5 QED	A	4
	<div style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;">•</div>	
6 QED	B	4
4 Cnj	$A \wedge B$	3
3 PE	$(A \wedge B) \vee C$	2
	C	(8)
	<div style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;">$\neg (A \wedge B)$</div>	
	•	
8 QED	C	7
7 PE	$(A \wedge B) \vee C$	2
2 PC	$(A \wedge B) \vee C$	

g.	$A \vee B$	1
	C	(5),(9)
	A	(4)
	<div style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;">$\neg (B \wedge C)$</div>	
	•	
4 QED	A	3
	•	
5 QED	C	3
3 Cnj	$A \wedge C$	2
2 PE	$(A \wedge C) \vee (B \wedge C)$	1
	B	(8)
	<div style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;">$\neg (A \wedge C)$</div>	
	•	
8 QED	B	7
	•	
9 QED	C	7
7 Cnj	$B \wedge C$	6
6 PE	$(A \wedge C) \vee (B \wedge C)$	1
1 PC	$(A \wedge C) \vee (B \wedge C)$	

h.

	$A \vee B$	1
	$\neg A \vee C$	2
	A	(5)
	$\neg A$	(5)
	$\neg B$	
	$\neg C$	
5 Nc	\perp	4
4 IP	C	3
3 PE	$B \vee C$	2
	C	(7)
	$\neg B$	
	\bullet	
7 QED	C	6
6 PE	$B \vee C$	2
2 PC	$B \vee C$	1
	B	(9)
	$\neg C$	
	\bullet	
9 QED	B	8
8 PE	$B \vee C$	1
1 PC	$B \vee C$	

i.

A	(3),(7)
$\neg(A \wedge B)$	5
\bullet	
A	2
3 QED	
B	(8)
\bullet	
A	6
7 QED	
\bullet	
B	6
8 QED	
A \wedge B	5
6 Cnj	
\perp	4
5 CR	
$\neg B$	2
4 RAA	
A \wedge $\neg B$	1
2 Cnj	
1 PE	
	(A \wedge B) \vee (A \wedge \neg B)

(A \wedge B) \vee (A \wedge \neg B)	1
A \wedge B	2
2 Ext	
A	(3)
2 Ext	
\bullet	
3 QED	
A	1
A \wedge \neg B	4
4 Ext	
A	(5)
4 Ext	
$\neg B$	
\bullet	
5 QED	
A	1
1 PC	
	A

2. a.

A \vee A	1	A	(2)
A	(2)	$\neg A$	
\bullet		\bullet	
2 QED		A	1
A	(3)	1 PE	
\bullet		A \vee A	
3 QED			
A	1		
1 PC			
	A		

b.

<table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-right: 1px solid black; padding: 5px;">$A \vee B$</td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> A</td><td style="padding: 5px;">(3)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> $\neg B$</td><td style="padding: 5px;"></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> •</td><td style="padding: 5px;"></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> A</td><td style="padding: 5px;">2</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> $B \vee A$</td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> B</td><td style="padding: 5px;"></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> $\neg A$</td><td style="padding: 5px;">(5)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> •</td><td style="padding: 5px;"></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> B</td><td style="padding: 5px;">4</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> $B \vee A$</td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> $B \vee A$</td><td style="padding: 5px;"></td></tr> </table>	$A \vee B$	1	A	(3)	$\neg B$		•		A	2	$B \vee A$	1	B		$\neg A$	(5)	•		B	4	$B \vee A$	1	$B \vee A$		<table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-right: 1px solid black; padding: 5px;">$B \vee A$</td><td style="padding: 5px;">2</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> $\neg A$</td><td style="padding: 5px;">(5)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> B</td><td style="padding: 5px;">(3)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> •</td><td style="padding: 5px;"></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> B</td><td style="padding: 5px;">2</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> A</td><td style="padding: 5px;">(5)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> $\neg B$</td><td style="padding: 5px;"></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> •</td><td style="padding: 5px;"></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> \perp</td><td style="padding: 5px;">4</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> B</td><td style="padding: 5px;">2</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> B</td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> $A \vee B$</td><td style="padding: 5px;"></td></tr> </table>	$B \vee A$	2	$\neg A$	(5)	B	(3)	•		B	2	A	(5)	$\neg B$		•		\perp	4	B	2	B	1	$A \vee B$	
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A	(3)																																																
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<p>3 QED</p> <p>2 PE</p> <p>5 QED</p> <p>4 PE</p> <p>1 PC</p>	<p>3 QED</p> <p>5 Nc</p> <p>4 IP</p> <p>2 PC</p> <p>1 PE</p>																																																

c.

<table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-right: 1px solid black; padding: 5px;">$(A \vee B) \vee C$</td><td style="padding: 5px;">3</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> $\neg A$</td><td style="padding: 5px;">(6)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> $\neg B$</td><td style="padding: 5px;">(8)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> $A \vee B$</td><td style="padding: 5px;">4</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> A</td><td style="padding: 5px;">(6)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> $\neg C$</td><td style="padding: 5px;"></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> •</td><td style="padding: 5px;"></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> \perp</td><td style="padding: 5px;">5</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> C</td><td style="padding: 5px;">4</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> B</td><td style="padding: 5px;">(8)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> $\neg C$</td><td style="padding: 5px;"></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> •</td><td style="padding: 5px;"></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> \perp</td><td style="padding: 5px;">7</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> C</td><td style="padding: 5px;">4</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> C</td><td style="padding: 5px;">3</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> C</td><td style="padding: 5px;">(9)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> •</td><td style="padding: 5px;"></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> C</td><td style="padding: 5px;">3</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> C</td><td style="padding: 5px;">2</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> $B \vee C$</td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;"> $A \vee (B \vee C)$</td><td style="padding: 5px;"></td></tr> </table>	$(A \vee B) \vee C$	3	$\neg A$	(6)	$\neg B$	(8)	$A \vee B$	4	A	(6)	$\neg C$		•		\perp	5	C	4	B	(8)	$\neg C$		•		\perp	7	C	4	C	3	C	(9)	•		C	3	C	2	$B \vee C$	1	$A \vee (B \vee C)$		<p>This is the second of the two derivations needed; the first appears in 4.2.3. In that one, disjunctive resources are exploited before disjunctive goals are planned for while the derivation at the left here illustrates the opposite approach.</p>
$(A \vee B) \vee C$	3																																										
$\neg A$	(6)																																										
$\neg B$	(8)																																										
$A \vee B$	4																																										
A	(6)																																										
$\neg C$																																											
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\perp	5																																										
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$A \vee (B \vee C)$																																											
<p>6 Nc</p> <p>5 IP</p> <p>8 Nc</p> <p>7 IP</p> <p>4 PC</p> <p>9 QED</p> <p>3 PC</p> <p>2 PE</p> <p>1 PE</p>	<p>5</p> <p>4</p> <p>(8)</p> <p>7</p> <p>4</p> <p>3</p> <p>(9)</p> <p>3</p> <p>2</p> <p>1</p>																																										

d.

	$A \vee (B \wedge \neg B)$	1		A		(2)
	A	(2)		$\neg (B \wedge \neg B)$		
	•			•		
2 QED	A	1	2 QED	A		1
	$B \wedge \neg B$	3	1 PE	$A \vee (B \wedge \neg B)$		
3 Ext	B	(5)				
3 Ext	$\neg B$	(5)				
	$\neg A$					
	•					
5 Nc	\perp	4				
4 IP	A	1				
1 PC	A					

e.

	$\neg (A \vee B)$	3,7		$\neg A \wedge \neg B$	1
	A	(5)	1 Ext	$\neg A$	(4)
	$\neg B$		1 Ext	$\neg B$	(5)
	•			$A \vee B$	3
5 QED	A	4		A	(4)
	•			•	
4 PE	$A \vee B$	3	4 Nc	\perp	3
3 CR	\perp	2		B	(5)
2 RAA	$\neg A$	1		•	
	B	(9)		\perp	3
	$\neg A$		3 PC	\perp	2
	•		2 RAA	$\neg (A \vee B)$	
9 QED	B	8			
8 PE	$A \vee B$	7			
7 CR	\perp	6			
6 RAA	$\neg B$	1			
1 Cnj	$\neg A \wedge \neg B$				

f.

	$\neg(A \wedge B)$	3		$\neg A \vee \neg B$	3
	A	(5)		A \wedge B	2
	B	(6)	2 Ext	A	(4)
	•		2 Ext	B	(5)
5 QED	A	4		$\neg A$	(4)
	•			⊥	3
6 QED	B	4	4 Nc	$\neg B$	(5)
	A \wedge B	3		•	
4 Cnj	⊥	2		⊥	3
3 CR	$\neg B$	1	5 Nc	⊥	3
2 RAA	$\neg A \vee \neg B$		3 PC	⊥	1
1 PE	$\neg(A \wedge B)$		1 RAA	$\neg(A \wedge B)$	

3. a.

	A \vee B	2		A B	
	A			A \vee B, A / \neg B	
	B			T T	Ⓣ Ⓣ Ⓣ
	A			⊥	2
	⊙	$A, B \not\Rightarrow \perp$		⊥	2
	⊥	2		B	
	⊙	$A, B \not\Rightarrow \perp$		⊥	2
	⊥	2		⊥	1
2 PC	$\neg B$			$\neg B$	
1 RAA	$\neg B$			$\neg B$	

b.

	$A \vee (B \wedge C)$	3,8	
	$\neg A$	(5)	
	A	(5)	
	$\neg B$		
	•		
	\perp	4	
5 Nc	\perp		
4 IP	B	3	
	$B \wedge C$		
6 Ext	B	7	
6 Ext	C		
	•		
7 QED	B	3	
3 PC	B	2	
2 PE	$A \vee B$	1	
	A		
	$\neg C$		
	\circ	$A, \neg C \not\Rightarrow \perp$	
	\perp	9	
9 IP	C	8	
	$B \wedge C$	10	
10 Ext	B		
10 Ext	C	11	
	•		
11 QED	C	8	
8 PC	C	1	
1 Cnj	$(A \vee B) \wedge C$		

Since entailment fails in one direction, equivalence must fail, so a second derivation for entailment in the other direction need not be pursued; but that entailment does hold, as is shown below.

	$(A \vee B) \wedge C$	1	
1 Ext	$A \vee B$	2	
1 Ext	C	(8)	
	A	(4)	
	$\neg (B \wedge C)$		
	•		
4 QED	A	3	
3 PE	$A \vee (B \wedge C)$	2	
	B	(7)	
	$\neg A$		
	•		
7 QED	B	6	
	•		
8 QED	C	6	
6 Cnj	$B \wedge C$	5	
5 PE	$A \vee (B \wedge C)$	2	
2 PC	$A \vee (B \wedge C)$		

Each of the following divides the one open gap:

A	B	C	$A \vee (B \wedge C)$	$/$	$(A \vee B) \wedge C$
T	T	F	Ⓣ	F	T
T	F	F	Ⓣ	F	T

c.

	$\neg(A \vee B)$	3
	A	(5)
	B	
	$\neg B$	
	•	
5 QED	A	4
4 PE	$A \vee B$	3
3 CR	\perp	2
2 RAA	$\neg B$	1
1 PE	$\neg A \vee \neg B$	

	$\neg A \vee \neg B$	2
	$A \vee B$	3.5
	$\neg A$	(4)
	A	(4)
	•	
4 Nc	\perp	3
	B	
	○	$\neg A, B \Rightarrow \perp$
	\perp	3
3 PC	\perp	2
	$\neg B$	(6)
	A	
	○	$A, \neg B \Rightarrow \perp$
	\perp	5
	B	(6)
	•	
6 Nc	\perp	5
5 PC	\perp	2
2 PC	\perp	1
1 RAA	$\neg(A \vee B)$	

The following divide the first and second open gap, respectively:

A	B	$\neg A \vee \neg B$	$\neg B$	$\neg(A \vee B)$	
F	T	T	Ⓟ	F	Ⓟ
T	F	F	Ⓟ	T	T