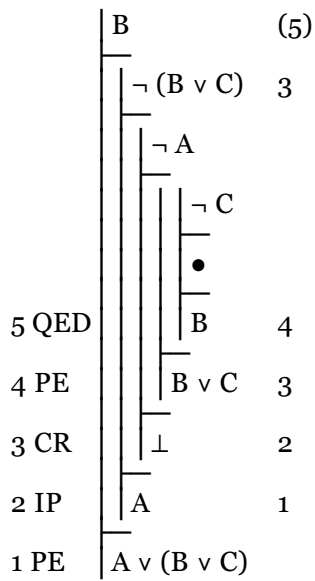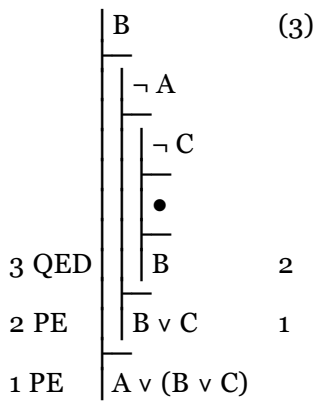## 4.2.3. Further examples

Both disjunction rules are illustrated by the derivation at the right, in which one grouping of a three-part disjunction is shown to entail the other. Choices between the two ways of planning for a goal disjunction were made at stages 2, 3, 5, 6, and 7 in accordance with the rules of thumb given above. Each choice helped to shorten the derivation—though only by a few steps. The derivation is contrived to provide several examples of this rule; we might have instead planned for initial the goal at stage 1 before exploiting the premise rather than planning for it separately in each of three gaps.

```
              │ A ∨ (B ∨ C)         1
              ├─────────────
              │ │ A                 (4)
              │ ├─────
              │ │ │ ¬ C
              │ │ ├───
              │ │ │ │ ¬ B
              │ │ │ ├───
              │ │ │ │ •
              │ │ │ ├───
   4 QED      │ │ │ A               3
   3 PE       │ │ A ∨ B             2
   2 PE       │ (A ∨ B) ∨ C         1
              │ │ B ∨ C             5
              │ ├─────
              │ │ │ B               (8)
              │ │ ├───
              │ │ │ │ ¬ C
              │ │ ├───
              │ │ │ │ ¬ A
              │ │ │ ├───
              │ │ │ │ •
              │ │ │ ├───
   8 QED      │ │ │ B               7
   7 PE       │ │ A ∨ B             6
   6 PE       │ (A ∨ B) ∨ C         5
              │ │ C                 (10)
              │ ├───
              │ │ │ ¬ (A ∨ B)
              │ │ ├─────────
              │ │ │ •
              │ │ ├───
  10 QED      │ │ C                 9
   9 PE       │ (A ∨ B) ∨ C         5
   5 PC       │ (A ∨ B) ∨ C         1
   1 PC       (A ∨ B) ∨ C
```

The scale of the difference you can expect a choice between the two forms of PE to make is illustrated by the two derivations below.

	B	(3)
	¬ A	
	¬ C	
	•	
3 QED	B	2
2 PE	B v C	1
1 PE	A v (B v C)	

	B	(5)
	¬ (B v C)	3
	¬ A	
	¬ C	
	•	
5 QED	B	4
4 PE	B v C	3
3 CR	⊥	2
2 IP	A	1
1 PE	A v (B v C)	

Each chooses a different way of planning for the initial goal at stage 1. Notice that in the second, which makes the less efficient choice, we are led back to the goal B ∨ C in a couple of stages.

Glen Helman 25 Aug 2005