

3.4.xa. Exercise answers

1. a.

	$\neg B$	
	$A \wedge \neg B$	2
2 Ext	A	
2 Ext	$\neg B$	
	○	$A, \neg B \Rightarrow \perp$
	⊥	1
1 RAA	$\neg (A \wedge \neg B)$	
$A \ B$	$\neg B / \neg (A \wedge \neg B)$	
T F	Ⓣ	Ⓣ
T F	Ⓣ	Ⓣ

b.

	$\neg (A \wedge B)$	3,8
	A	(5)
	●	
5 QED	A	4
	$\neg B$	
	○	$A, \neg B \Rightarrow \perp$
	⊥	6
6 IP	B	4
4 Cnj	$A \wedge B$	3
3 CR	⊥	2
2 RAA	$\neg A$	1
	B	(11)
	$\neg A$	
	○	$\neg A, B \Rightarrow \perp$
	⊥	10
10 IP	A	9
	●	
11 QED	B	9
9 Cnj	$A \wedge B$	8
8 CR	⊥	7
7 RAA	$\neg B$	1
1 Cnj	$\neg A \wedge \neg B$	
$A \ B$	$\neg (A \wedge B) / \neg A \wedge \neg B$	
T F	Ⓣ	F F Ⓣ
F T	Ⓣ	F T Ⓣ

divides the first dead-end gap

divides the second dead-end gap

c.

	$\neg(A \wedge B)$	3			
	$\neg(B \wedge C)$	7			
	$A \wedge C$	2			
2 Ext	A				
2 Ext	C		(10)		
	\bullet				
5 QED	A	4			
	$\neg B$				
	$\neg B$				
	\circ		A, $\neg B$, C $\neq \perp$		
	\perp	9			
9 IP	B	8			
	\bullet				
10 QED	C	8			
8 Cnj	B \wedge C	7			
7 CR	\perp	6			
6 IP	B	4			
4 Cnj	A \wedge B	3			
3 CR	\perp	1			
1 RAA	$\neg(A \wedge C)$				
A B C $\neg(A \wedge B)$, $\neg(B \wedge C)$ / $\neg(A \wedge C)$					
T F T \textcircled{T} F \textcircled{T} F \textcircled{F} T					

2. a.

	$\neg(A \wedge \neg B)$	2			
	$\neg B$	(5)			
	$\neg A$				
	\circ		$\neg A$, $\neg B \neq \perp$		
	\perp	4			
4 IP	A	3			
	\bullet				
5 QED	$\neg B$	3			
3 Cnj	A \wedge $\neg B$	2			
2 CR	\perp	1			
1 RAA	B				
A B $\neg(A \wedge \neg B)$ / B					
F F \textcircled{T} F T \textcircled{F}					

b.

	$\neg (A \wedge B)$	3
	$B \wedge A$	2
2 Ext	B	(6)
2 Ext	A	(5)
		•
5 QED	A	4
		•
6 QED	B	4
4 Cnj	$A \wedge B$	3
3 CR	\perp	1
1 RAA	$\neg (B \wedge A)$	

c.

	$\neg (A \wedge \neg B)$	3
	$B \wedge \neg A$	2
2 Ext	B	
2 Ext	$\neg A$	
		$\neg A$
		○
		$\neg A, B \Rightarrow \perp$
	\perp	5
5 IP	A	4
		B
		○
		$\neg A, B \Rightarrow \perp$
	\perp	6
6 IP	$\neg B$	4
4 Cnj	$A \wedge \neg B$	3
3 CR	\perp	1
1 RAA	$\neg (B \wedge \neg A)$	

A	B	$\neg (A \wedge \neg B)$	/	$\neg (B \wedge \neg A)$
F	T	⊙	F	F
F	F	⊙	F	⊙
T	T	⊙	T	T

d.

	$\neg(A \wedge B)$	3,8	
	$\neg(B \wedge C)$	11	
	B	(6),(10),(14)	

	A	(5)	

5 QED	A	4	

	•		

6 QED	B	4	

4 Cnj	A \wedge B	3	

3 CR	\perp	2	

2 RAA	$\neg A$	1	

	C	(15)	

	$\neg A$		

	$\neg A$		

	•		

14 QED	B	13	

	•		

15 QED	C	13	

13 Cnj	B \wedge C	12	

12 CR	\perp	11	

11 IP	A	9	

	•		

10 QED	B	9	

9 Cnj	A \wedge B	8	

8 CR	\perp	7	

7 RAA	$\neg C$	1	

1 Cnj	$\neg A \wedge \neg C$		

	$\neg(A \wedge B)$	3
	$\neg(B \wedge C)$	8
	B	(6),(10)

	A	(5)

5 QED	A	4

	•	

6 QED	B	4

4 Cnj	A \wedge B	3

3 CR	\perp	2

2 RAA	$\neg A$	1

	C	(11)

	•	

10 QED	B	9

	•	

11 QED	C	9

9 Cnj	B \wedge C	8

8 CR	\perp	7

7 RAA	$\neg C$	1

1 Cnj	$\neg A \wedge \neg C$	

The derivation on the left exploits resources in their order of appearance; while the one above chooses, at stage 8, the resource that is most closely connected with other resources of the gap in which it is exploited. Notice that derivation on the left is eventually led to exploit the same resource to the same effect.

e.

	$\neg (A \wedge \neg (B \wedge \neg (C \wedge \neg D)))$	3													
	$A \wedge \neg (B \wedge D)$	2													
2 Ext	A	(5)													
2 Ext	$\neg (B \wedge D)$	8													
	•														
5 QED	A	4													
	$B \wedge \neg (C \wedge \neg D)$	7													
7 Ext	B	(10)													
7 Ext	$\neg (C \wedge \neg D)$	12													
	•														
10 QED	B	9													
	$\neg D$	(15)													
	$\neg C$														
	○		A, B, $\neg C$, $\neg D \not\Rightarrow \perp$												
	\perp	14													
14 IP	C	13													
	•														
15 QED	$\neg D$	13													
13	C $\wedge \neg D$	12													
12 CR	\perp	11													
11 IP	D	9													
9 Cnj	B $\wedge D$	8													
8 CR	\perp	6													
6 RAA	$\neg (B \wedge \neg (C \wedge \neg D))$	4													
4 Cnj	A $\wedge \neg (B \wedge \neg (C \wedge \neg D))$	3													
3 CR	\perp	1													
1 RAA	$\neg (A \wedge \neg (B \wedge D))$														
A B C D		$\neg (A \wedge \neg (B \wedge \neg (C \wedge \neg D))) / \neg (A \wedge \neg (B \wedge D))$													
T	T	F	F	Ⓣ	F	F	T	T	F	T	F	Ⓣ	T	T	F