

2.4.s. Summary

The introduction of a lemma is one way of dividing up the work of a proof. We can implement this idea in derivations by dividing a gap into two, one with the lemma as a goal and the other with it as a further assumption to use in reaching the goal of the parent gap. The rule [Lemma \(Lem\)](#) that does this is not safe in general nor is it always progressive, and we will use only special instances of it.

A lemma is always safe when it is entailed by the goal it is designed to help us reach. The principal use of this idea will come in arguments whose goal is \perp —that is, in [reductio arguments](#). Since \perp entails any sentence a rule [Lemma for Reductio \(LFR\)](#) which allows free use of lemmas in *reductio* arguments will be safe (though some restriction on its use is needed to insure it is progressive).

A lemma is also safe if we know we can reach it. Rules applying this idea will be designed for particular sorts of entailment and, since the lemma is known to follow from our resources, there is no need to divide the gap or even introduce a new scope line. Indeed, we will use this sort of lemma only in [attachment rules](#) that add the lemma as an available but inactive resource. The first example of this sort of rule is [Adjunction \(Adj\)](#) which adds a conjunction when both conjuncts are already available. Although attachment rules can help us to close gaps sooner, the rules themselves are not direct so some care is needed in their use if they are to be progressive.

The derivation rules we have so far are summarized in the table below. The names of the rules are links to the point in the text where they were initially described; look there to see the actual form taken by the rule.

Rules for developing gaps

for resources for goals

conjunction

$\varphi \wedge \psi$

Ext

Cnj

Rules for closing gaps

when to close rule

the goal is also

a resource

QED

\top is the goal

ENV

\perp is a resource

EFQ

Basic system

Attachment rule

added resource rule

$\varphi \wedge \psi$

Adj

*Added rules
(optional)*