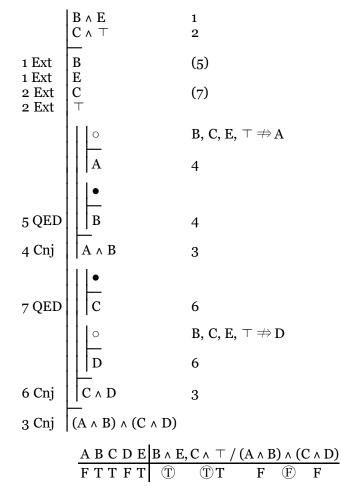
2.3.xa. Exercise answers

1. (2)  $2 \text{ QED} \begin{vmatrix} \bullet \\ A & 1 \\ 0 & A \neq B \\ B & 1 \end{vmatrix}$ 1 Cnj  $A \wedge B$  $\begin{array}{c|c} A & B & A / A \land B \\ \hline T & F & \hline \end{array}$ 2.  $A \land B$ 1 A B (4),(6) (5) 1 Ext 1 Ext 4 QED A 2 B 5 QED 3 6 QED 3 3 Cnj B∧A 2 2 Cnj  $| A \land (B \land A) |$ 



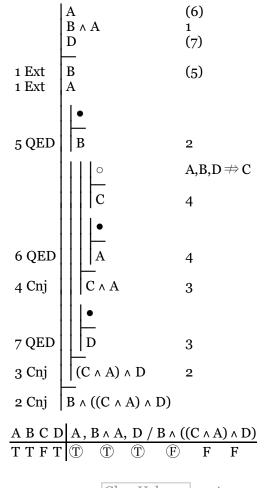
The derivation could have been ended after stage 4 when the first open gap has reached a dead end. Often answers will show a derivation continued further than necessary in order to show how the further steps would have worked out. The counterexample presented here divides both dead-end gaps; there are others that divide one of the two. Notice that  $\top$  is not assigned a value at the left of the table. Since its value is fixed by the stipulation that it is a tautology, a value need not and cannot be assigned to it as part of an extensional interpretation.

3.

	$\begin{array}{c} A \land B \\ B \land C \\ B \land D \end{array}$	1 2 3
1 Ext 1 Ext 2 Ext 2 Ext	A B B C	(5)
3 Ext 3 Ext	B D	(6)
5 QED	•   A	4
6 QED	D	4
4 Cnj	A ^ D	

Clearly, there is redundancy in the active resources of the gaps after stage 3. Since both gaps close, the exploitation of the second premise at stage 2 is not necessary (though it would be necessary before any gap could reach a dead end). It would be possible to state rules so that the resource B was not repeated at stages 2 and 3, but such repetition does not ordinarily enlarge derivations significantly and makes it easier to check whether rules have been applied fully and correctly.

4.



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5.