2.3.xa. Exercise answers

1.

	A	(2)
	•	
2 QED	A	1
	○	A ⇏ B
	B	1
1 Cnj	A ∧ B	

A	B	A / A ∧ B
T	F	Ⓣ Ⓕ

2.

	A ∧ B	1
1 Ext	A	(4),(6)
1 Ext	B	(5)
	•	
4 QED	A	2
	•	
5 QED	B	3
	•	
6 QED	A	3
3 Cnj	B ∧ A	2
2 Cnj	A ∧ (B ∧ A)	

3.

	$B \wedge E$	1
	$C \wedge \top$	2
1 Ext	B	(5)
1 Ext	E	
2 Ext	C	(7)
2 Ext	$\top$	
	○	B, C, E, $\top \Rightarrow$ A
	A	4
	●	
5 QED	B	4
4 Cnj	$A \wedge B$	3
	●	
7 QED	C	6
	○	B, C, E, $\top \Rightarrow$ D
	D	6
6 Cnj	$C \wedge D$	3
3 Cnj	$(A \wedge B) \wedge (C \wedge D)$	

A	B	C	D	E	$B \wedge E, C \wedge \top$ / $(A \wedge B) \wedge (C \wedge D)$
F	T	T	F	T	Ⓣ Ⓣ T F Ⓕ F

The derivation could have been ended after stage 4 when the first open gap has reached a dead end. Often answers will show a derivation continued further than necessary in order to show how the further steps would have worked out. The counterexample presented here divides both dead-end gaps; there are others that divide one of the two. Notice that $\top$ is not assigned a value at the left of the table. Since its value is fixed by the stipulation that it is a tautology, a value need not and cannot be assigned to it as part of an extensional interpretation.

4.

	A ∧ B	1
	B ∧ C	2
	B ∧ D	3
	———	
1 Ext	A	(5)
1 Ext	B	
2 Ext	B	
2 Ext	C	
3 Ext	B	
3 Ext	D	(6)
	| •	
	| ———	
5 QED	| A	4
	| •	
	| ———	
6 QED	| D	4
	———	
4 Cnj	A ∧ D	

Clearly, there is redundancy in the active resources of the gaps after stage 3. Since both gaps close, the exploitation of the second premise at stage 2 is not necessary (though it would be necessary before any gap could reach a dead end). It would be possible to state rules so that the resource B was not repeated at stages 2 and 3, but such repetition does not ordinarily enlarge derivations significantly and makes it easier to check whether rules have been applied fully and correctly.

5.

	A	(6)
	B ∧ A	1
	D	(7)
	⊢	
1 Ext	B	(5)
1 Ext	A	
	⊢•	
5 QED	B	2
	⊢○	A,B,D ⇏ C
	C	4
	⊢•	
6 QED	A	4
4 Cnj	C ∧ A	3
	⊢•	
7 QED	D	3
3 Cnj	(C ∧ A) ∧ D	2
2 Cnj	B ∧ ((C ∧ A) ∧ D)	

A	B	C	D	A , B ∧ A, D / B ∧ ((C ∧ A) ∧ D)					
T	T	F	T	Ⓣ	Ⓣ	Ⓣ	Ⓕ	F	F

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