2.3.3. Presenting counterexamples

A dead-end open gap is always divided by an interpretation that also divides the ultimate argument of the derivation, and we will complete derivations that uncover invalidity by displaying this division. We will do that by exhibiting the interpretation that divides a dead-end open gap and calculating the truth values of the original premises and conclusions on that interpretation. In the example discussed in 2.3.1, this calculation is shown in the following table:

A	B	C	$(A \wedge \top) \wedge B$ / $B \wedge (\top \wedge C)$
T	T	F	T T Ⓣ Ⓕ T F

Here the values of unanalyzed components have not been repeated on the right, but they are used to calculate the values of compounds containing them, with the order of calculation being guided by parentheses. In performing this calculation we are confirming that the interpretation dividing the gap really does constitute a counterexample to the ultimate argument; and we will say that, in constructing the table, we are ***presenting a counterexample***. It will be our standard way of concluding the treatment of an argument whose derivation fails.

It is not always the case that the unanalyzed components of the ultimate argument all appear among the resources and goal of a dead-end gap. When unanalyzed components do not appear there, values must still be assigned to them in order for a truth value to be defined for each sentence in the ultimate argument; but it will not matter what value we assign to these further unanalyzed components. If an interpretation divides the gap, any way we choose to extend it to unanalyzed components not appearing in the gap's proximate argument will still divide that gap and therefore divide the ultimate argument. The following example is designed to illustrate this.

	A ∧ B	1
1 Ext	A	
1 Ext	B	(4)
	○	A, B ⇏ C
	C	2
	●	
4 QED	B	3
	○	A, B ⇏ D
	D	3
3 Cnj	B ∧ D	2
2 Cnj	C ∧ (B ∧ D)	

A B C D	A ∧ B	/ C ∧	(B ∧ D)	
T T F T	Ⓣ	Ⓕ	T	divides first dead-end gap
T T F F	Ⓣ	Ⓕ	F	divides both dead-end gaps
T T T F	Ⓣ	Ⓕ	F	divides second dead-end gap

Of the three interpretations shown, the first divides only the first dead-end gap (since it assigns the value **T** to the goal of the second dead-end gap), and the last divides only the second open gap (for a similar reason); but the middle one divides both open gaps. With 4 unanalyzed components, there are $2{\times}2{\times}2{\times}2 = 2^4{=}16$ possible interpretations, so there are 13 interpretations that do not divide either gap. The soundness and safety of our rules insures that the 3 interpretations shown above constitute counterexamples to the ultimate argument and that the other 13 do not.

Any one of these three interpretations is enough to provide a counterexample, so any of them could be used to provide a counterexample. Beginning with chapter 6, it will prove to be most convenient to assign **F** to an unanalyzed component whenever we have a choice, and here that would lead us to the middle interpretation in the case of both gaps. But, for now, when an unanalyzed component does not appear in the proximate argument of a dead-end gap, the choice of the value to assign to it is entirely arbitrary.

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