2.2.6. Resources

The ideas of available and active resources have been used at several points already, but they have not yet been explained fully. A resource counts as ***available*** in a gap if it was entered either as one of the initial premises of the derivation or in the course of developing the gap in question. The system of scope lines can be used to tell which resources are available in a gap: a resource is available if every scope line to its left continues unbroken at the left of the gap. On way of thinking about this is to suppose that each scope line indicates the left side of a box, as shown in Figure 2.2.6-1 and that a resource is available only to the gaps that are also within the smallest box containing it.

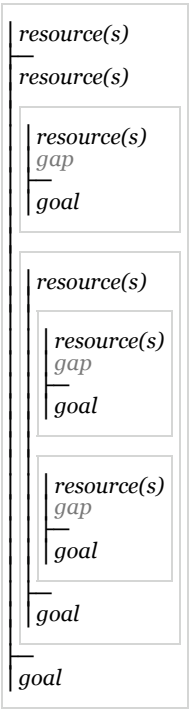
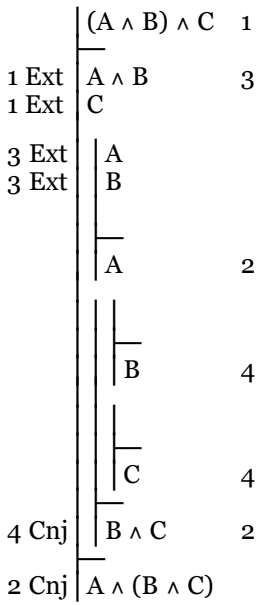


Fig. 2.2.6-1. The boxes indicated by the scope lines of a derivation. If JavaScript is enabled on the browser you are using, moving the cursor over a resource will color the gaps in which it is available green and shade areas where it is unavailable. Moving the cursor over a gap will color resources available in it green and shade areas whose resources are unavailable to it. The resource or gap that the cursor is over will be colored blue and underlined.

A resource is ***active*** in a gap if it is available in that gap and has not already been exploited in narrowing it. The easiest way to locate the

active resources of a gap is to scan the available resources and eliminate the inactive ones. To be inactive in any gap, a resource must have been exploited at some stage. If it has, there will be an unparenthesized stage number to its right. A resource may have been exploited only in some gaps and may still remain active in others. To be inactive in a given gap, the resource must have been exploited in narrowing the gap. To see whether this is so, we need to check all resources and goals that were introduced at a stage when the resource was exploited (i.e., at a stage whose number appear unparenthesized to the resource's right). (So far, we have seen goals introduced only in the course of planning for more distant goals, but in later chapters they will be introduced as part of the exploitation of certain resources.) If any such resource or goal is such that the smallest box containing it also contains the gap we are considering, it was introduced in the course of developing the gap. A resource may be exploited more than once, so there may be several stage numbers you will need to check. If any of them was a stage in which the gap you are considering was developed, the resource is no longer among the active resources of the gap.

This is illustrated by the partially developed derivation shown below. The three steps at the top of the derivation are resources available for each of the derivation's three gaps. The first, $(A \wedge B) \wedge C$, is inactive in all three gaps. It was exploited at stage 1, and that was the initial stage of development for all the gaps of the derivation. The second resource, $A \wedge B$, is inactive for the first of the gaps (having been exploited at stage 3 in developing this gap), but it is active for the remaining two gaps since the resources introduced at stage 3 did nothing to narrow these gaps (as is shown by the fact that the gaps are outside the smallest box surrounding the resources with 3 at their left). The third resource C has not been exploited at all (and could not be since it is not a conjunction), so it is active for all three gaps. Since the resource exploited at stage 3 must be exploited again in order to close the second gap, it would have been a little more efficient to exploit this resource before dividing the initial gap in two; but the derivation as shown is perfectly correct (though still unfinished).



One way of thinking about this is to suppose that a given gap can see only those parts of a derivation that are not boxed off from it—i.e., only those parts all of whose scope lines continue to the left of the gap. If a stage number appears at the left only in parts of the derivation that are invisible to the gap, it is also invisible—even when it appears to the right of resources that are visible. This idea is illustrated in Figure 2.2.6-2 below where the same derivation is shown from the perspective of each of the three gaps in turn. Material that is boxed off from a gap is shown in light gray. Notice that the number 3 at the right of the second line is invisible to the second and third gaps. As we saw earlier, that is because all the development at stage 3 is boxed off from the second and third gaps.

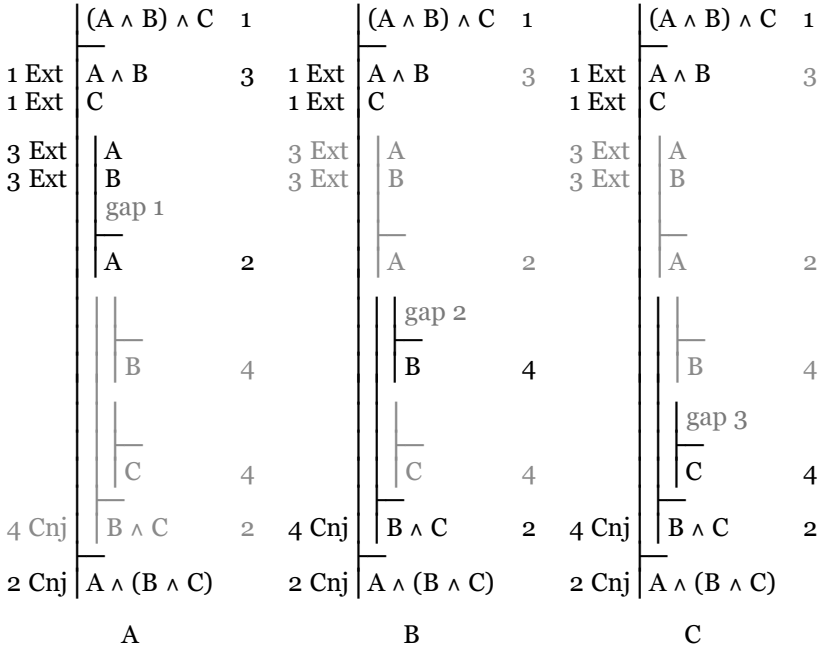
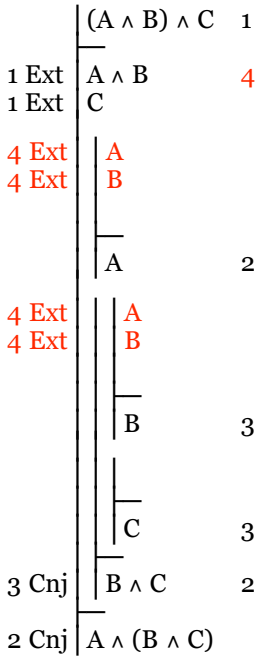


Fig. 2.2.6-2. A derivation from the perspective of each of its three gaps.

You can imagine a derivation to be the result of superimposing separate layers like these. There will be one layer for each gap with a gap's layer depicting its perspective on the derivation. When we distinguish the resources available for a gap or determine whether a resource has been used to narrow a gap, we are really considering that gap's layer separately.

When a gap is divided before a resource is exploited to narrow it, it is possible to exploit the resource to narrow several gaps at once. This is shown in the partial derivation below (which has the same initial premises and conclusion as the one we have been considering).



In this derivation, one of the resources has just been exploited at stage 4 to narrow two different gaps. Thereafter, it is inactive in these gaps but still active in the third (where it happens to be unneeded). Some of the resources added at stage 4 will be invisible to each of the first two gaps; but, because other added resources are visible, the number 4 at the right is visible from both these gaps. However, none of the resources added at stage 4 is visible from the third gap, so the number 4 at the right is not visible from it.

Since we use a similar numerical notation for both resources that are exploited and goals that have been planned for, you might expect that the concepts of availability and activity can be applied to goals as well as resources; and, indeed, they can be. If we were to consider derivations for relative exhaustiveness, we would need to engage in the same sort of accounting for goals that we have been considering for resources. However, in a system of derivations for entailment alone like the one we will actually use, each gap has just one active goal, which appears just below the gap. Goals at earlier stages of a gap's development (i.e., the goals that are not boxed off from the gap) could be described as "available", but they are not available for any sort of use. In particular, although we can consider all available resources when looking for a way of closing a gap, it is only the active goal and not any earlier one that we consider. (Some of the arguments of 2.3.2 could be used to show that

considering all “available” goals would not lead us to count an invalid argument as valid, but looking at derivations in this way would make them less like the patterns of ordinary explicit deductive argumentation, which seem to be focused always on a single conclusion.)

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