2.1.7. Interpretations

In passing from a sentence to any of its forms, we abstract from the specific sentences that we replace by variables. In general, we also abstract from the proposition expressed by the sentence and from its truth value. Except in special cases, such as forms that are shared only by tautologies, a logical form does not express a proposition or have a truth value, but we may introduce such semantic features by ***interpreting*** the form.

We will consider two sorts of interpretation, an ***extensional interpretation***, that provides a truth value only, and an ***intensional interpretation***, which provides the proposition expressed and thus a truth value not only for the actual world but for every possible world. These two sorts of interpretation will be used for different purposes, so it will usually be clear from the context which sort is relevant; and, when this is clear, we will use the term ***interpretation*** without qualification.

The term ***intensional*** (spelled with an *s*) and the term ***extensional*** derive from a traditional distinction between, on the one hand, the means by which a term picks out a class of objects and, on the other, the class of objects it picks out. Terms that pick out the same class of objects in different ways have the same ***extension*** but different ***intensions***. For example, if the population of Crawfordsville is 14287, the terms *city with a population greater than 14287* and *city more populous than Crawfordsville* have the same extension but different intensions. One way to see that the two terms have different intensions is to notice that they would pick out different classes of cities if the population of Crawfordsville were not 14287.

During the past century, the concepts of intension and extension have been extended to terms that pick out single objects rather than classes of objects, so we can say that the definite descriptions *the author of Poor Richard's Almanack* and *the inventor of the lightning rod* both have Benjamin Franklin as their extension though they differ in their intensions. The distinction between the object a term refers to and the way it refers to it is sufficiently analogous to the distinction between the truth value of a sentence and the proposition it expresses that the concepts of intension and extension are now also applied to sentences. So *Indianapolis is the*

capital of Indiana and *Sringfield is the capital of Illinois* could be said to have the same extension (i.e., the value **T**) but different intensions. In general, the extension of sentence is it the truth value while its intension is the proposition it expresses.

Since the only general way we have to specify propositions is by using sentences that express them, intensional interpretations will be specified by assigning sentences to variables. (This assumes were are working with a fixed context of use, so sentences express propositions.) This assignment is the exact inverse of the process of abbreviating ultimate components by capitals, and we will use the same notation for the association of letters and sentences that results. For example, we can give an intensional interpretation of the form (A ∧ B) ∧ C by making the following assignment of sentences to the variables that mark its ultimate components.

A: *I got it apart*;
B: *I don't know how I got it apart*;
C: *I couldn't get it together again*

Since the sentences assigned to variables serve only to specify propositions, we will not be concerned about their logical forms; they may be as simple or complex as we wish.

Especially in later chapters, the proposition assigned to a compound sentence by an intensional interpretation may not be apparent until we find an idiomatic English sentence that expresses the same proposition. This can be done by a step-by-step process of ***synthesizing*** English that reverses the process of analysis. For the example above, this might proceed as follows:

(*I got it apart* ∧ *I don't know how I got it apart*) ∧ *I couldn't get it together again*
I got it apart but I don't know how ∧ *I couldn't get it together again*
I got it apart but I don't know how, and I couldn't get it together again

Of course, other wording is possible here, and the process of synthesizing English will rarely have a unique correct result.

Extensional interpretations are easier to manage and will often provide all the information we need. The following illustrates a convenient notation for an assignment of truth values to variables:

A	B	C
T	F	T

We adapt the tabular notation used for truth tables, writing the variables left to right and the assigned value under each. The values of the larger components may be calculated by using the truth table for conjunction just as a multiplication table may be used to calculate the numerical value of a product: we find the values of the smallest components first and use these to calculate the values of larger components. The notation shown above can be extended for this purpose in the following way:

A	B	C	(A ∧ C) ∧ (B ∧ C)		
T	F	T	T	Ⓕ	F

The whole form we are interested in is displayed to the right of its ultimate components, and the truth value calculated for each compound component is displayed below the main connective of that component. The value for the sentence as a whole is shown circled. Our interest will generally be only in this final value, but examples in this text will usually show how it was reached by also displaying the intermediate values.

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