1.4.xa. Exercise answers

1. **a.** φ and ψ together entail χ

b. $\psi \Rightarrow \varphi$

c. φ is equivalent to itself

d. ψ is absurd
or: ψ taken by itself forms an inconsistent set

e. $\Gamma, \varphi \Rightarrow$
or: $\Gamma, \varphi \Rightarrow \bot$
(Strictly speaking, $\Gamma, \varphi \Rightarrow \bot$ expresses entailment rather than inconsistency, but it is true if and only if φ is inconsistent with Γ.)

f. $\Gamma, \psi \Rightarrow \varphi$

2. **a.** We have supposed that $\Gamma \Rightarrow \varphi$. That is, we have supposed that φ is **T** in any possible world in which all members of Γ are **T**. But w is a world in which all members of Γ are **T**, so φ, too, must be **T** in w.

b. We now know that φ and all members of Γ are **T** in w. But we supposed that $\Gamma, \varphi \Rightarrow \psi$ and we now know that all the premises of this entailment are **T** in w, so ψ also must be **T** also.

c. For w to be a counterexample to $\Gamma \Rightarrow \psi$, it must make give ψ the value **F** and give all the members of Γ the value **T**.

d. A counterexample to $\Gamma \Rightarrow \varphi$ must give φ the value **F** and give all the members of Γ the value **T**. A counterexample to $\Gamma, \varphi \Rightarrow \psi$ must give ψ the value **F** while giving φ and all the members of Γ the value **T**.

e. We know that w gives ψ the value **F** and gives all the members of Γ the value **T**. But it also must make φ either **T** or **F**. If it does the former, it is a counterexample to $\Gamma, \varphi \Rightarrow \psi$; and if it does the latter, it is a counterexample to $\Gamma \Rightarrow \varphi$.