

1.4.x. Exercise questions

1. Restate each of the following claims about logical properties and relations, putting into symbolic notation those stated in English and into English those stated in symbolic notation:
 - a. $\varphi, \psi \Rightarrow \chi$
 - b. φ is entailed by ψ
 - c. $\varphi \Leftrightarrow \varphi$
 - d. $\psi \Rightarrow$
 - e. φ is inconsistent with Γ
 - f. φ is entailed by the members of Γ together with ψ
2. The following steps lead you to construct a proof of the law for lemmas

if $\Gamma, \varphi \Rightarrow \psi$ and $\Gamma \Rightarrow \varphi$, then $\Gamma \Rightarrow \psi$

Begin by supposing that $\Gamma, \varphi \Rightarrow \psi$ and $\Gamma \Rightarrow \varphi$ are both true. We want to show that, under this supposition, $\Gamma \Rightarrow \psi$ is also true. To do that, we consider any possible world w in which all members of Γ are true and try to show that ψ is true in w .

- a. Our supposition that $\Gamma, \varphi \Rightarrow \psi$ and $\Gamma \Rightarrow \varphi$ are both true combined with what we know about w enables us to conclude that φ is true. Why?
- b. Adding the information that φ is true in Γ to what we already knew, we can conclude that ψ is true. Why?

So, knowing about w only that all members of Γ were true, we are able to conclude that ψ is true. And that shows us that ψ is true in every world in which all members of Γ are true, which means that $\Gamma \Rightarrow \psi$.

Another approach to proving the law is to show that $\Gamma \Rightarrow \psi$ fails only if at least one of $\Gamma, \varphi \Rightarrow \psi$ and $\Gamma \Rightarrow \varphi$ fails. The following three steps show this:

- c. Suppose that w is a counterexample to $\Gamma \Rightarrow \psi$. What truth values do ψ and the members of Γ have in w ?
- d. What truth values are needed to have a counterexample to $\Gamma \Rightarrow \varphi$? To have a counterexample to $\Gamma, \varphi \Rightarrow \psi$?

- e.** The world w from **c** will be a counterexample to either $\Gamma, \varphi \Rightarrow \psi$ or $\Gamma \Rightarrow \varphi$. Why?

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