

1.4.s. Summary

Entailment may be defined in two equivalent ways, either as the relation that holds when the conclusion is false in no possible world in which all the premises are true or as the relation which holds when the conclusion is true in all such worlds. The first approach can be stated more briefly by saying that an argument is valid when no world **divides** the premises from conclusion; a world that does divide premises from conclusion is a **counterexample** to the claim of entailment or validity.

The idea of entailment can also be understood by way of certain laws governing it. For example, if we limit ourselves to single-premised arguments—i.e., to implication—the relation is **reflexive** and **transitive**. The **law for premises** and the **chain law** are analogous principles that apply to entailment more generally. Entailment also obeys a principle of **monotonicity** asserting that a premises may always be added without undermining entailment (something does not hold for many forms of non-deductive inference) and a **law for lemmas** that tells us that a premise may be dropped when it is entailed by other premises.

Other properties and relations besides entailment can be given pairs of negative and positive definitions. This is true for the ideas of **logical equivalence** and **tautologousness** introduced in 1.2.2. Sentences are equivalent when they entail each other, and this **basic law** implies that equivalence is **symmetric** as well as **reflexive** and **transitive**. Moreover, equivalent statements may **replace** one another either as premises or conclusions of an argument without affecting its validity (unlike the case of entailment which obeys only the weaker laws of **conclusion covariance** and **premise contravariance**). The laws governing tautologies are most easily stated by focusing on the particular case of Tautology \top . For example, \top is **always a valid conclusion**, but it **never contributes anything as a premise** and may be **freely added to or dropped from the premises** without changing an argument's validity.

The definitions of **absurdity** are in a way opposite those of tautologousness and having **Absurdity \perp as a premise**, like having a \top as a conclusion, makes an argument valid. When an argument with \perp as its conclusion is valid, its premises form an **inconsistent**

set. Inconsistency is the fundamental negative concept of deductive logic and the relative concept of being excluded by or inconsistent with a set is a kind of negative opposite to entailment. As a relation between pairs of sentences relative inconsistency is symmetric and such sentences are said to be mutually exclusive. Although inconsistency is a fundamental deductive property, it is one we will establish by using laws that describe it in terms of entailment.

The negative concepts of inconsistency and exclusiveness are opposed in one way to entailment and in another way to exhaustiveness. Contradictory sentences are ones that are bound to differ in truth value; such sentences can be characterized as both mutually exclusive and jointly exhaustive. Exhaustiveness can be conditional and this is a relation between sets that generalizes entailment to allow a set of alternatives rather than a single conclusion. This relation fails when a possible world divides its premises from its alternatives by making the former all true and the latter all false. Relative exhaustiveness obeys cut law which are analogous to, but more symmetric than, the principles governing entailment.

Relative exhaustiveness has an important role in unifying the concepts of deductive logic. All the ones we have seen can be described as special cases of it. We can also use it to describe the absence of deductive properties and relations, whether this is the logical contingency of individual sentences or the logical independence of pairs or larger sets. Laws governing relative exhaustiveness in its own right tend to be symmetric in form. Relative exhaustiveness can be connected with entailment by law employing the idea of contradictoriness. This law exhibits a kind of symmetry that is found also in the laws for \top and \perp stated in terms of relative exhaustiveness. Their symmetry can also be seen as one instance of a relation of duality that we will encounter in other cases as well.