

### 1.2.s. Summary

The relation of entailment concerns the possibilities of truth and falsity for premises and conclusions; that is, it concerns the **truth values** of these sentences in various **possible worlds**. The possibilities in question are **logical possibilities**, which may be understood as those situations whose description is permitted by the semantic rules of the language.

Information about the truth values of a sentence in all possible worlds is information about its **truth conditions**, and these truth conditions determine the **proposition** it expresses. Sentences that have the same truth conditions, that express the same proposition, are **logically equivalent** (and an idea for which we use the sign  $\Leftrightarrow$ ). From the point of view of deductive logic, equivalent sentences have the same properties and stand in the same relations to other sentences. Entailment is one relation among sentences that depends only on the propositions they express. A conclusion  $\varphi$  is entailed by a set  $\Gamma$  of premises when  $\varphi$  rules out only worlds that are ruled out by at least one member of  $\Gamma$ . This is a way of saying that the content of  $\varphi$  does not exceed that of the members of  $\Gamma$  taken together, so entailment is a comparison of sentences in terms of their informational content. At one extreme are **tautologies**, which rule out no possibilities and thus have no content. All tautologies are equivalent and we will distinguish one, **Tautology**, for which we use the notation  $\top$ . At the other extreme are sentences that rule out all possibilities. Such sentences are **absurd** and all are equivalent to the single representative **Absurdity**, for which we use the notation  $\perp$ .

Although certain groups of sentences can be ordered linearly between  $\perp$  and  $\top$  as a series of claims with steadily increasing content, the full range of propositions expressed by sentences are better thought of as inhabiting a much more complex **logical space**. This can be thought of, on the one hand, as a space of possibilities, with an individual proposition constituting a division of the space into two regions, the possibilities it rules out and the possibilities it leaves open. Another sort of space has as its points not possible worlds but propositions, with different possible worlds representing different dimensions with respect to which the location of propositions can differ. Logical space in this sense has a

bottom in the proposition expressed by  $\perp$  and a top provided by  $\top$ . So long as there are alternative possibilities (that is, more than just one possible world), there will be more propositions with intermediate content than there are degrees of content intermediate between  $\perp$  and  $\top$ .

This picture of deductive reasoning fits into a simplified picture of the function of language. Our beliefs, the information we think we have, amount to a proposition that rules out a certain range of possibilities for the history of the universe. In general, we would like to narrow down the range of possibilities left open even further. When language is used cooperatively (something that must be the standard case), we share the ability to rule out possibilities by asserting sentences that rule out some of the possibilities our beliefs lead us to exclude. The sentences we can sincerely assert are the ones that express propositions that are entailed by the proposition expressing the sum total of our beliefs.

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