1.2.1. Truth values and possible worlds

When an inference is deductive—when its premises and conclusion constitute a valid argument—its conclusion cannot be in error unless there is an error somewhere in its premises. The sort of error in question lies in a statement being false, so to know that an argument is valid is to know that its conclusion must be true unless at least one premise is false. This means that the ideas of truth and falsity have a central place in our discussion of deductive logic, and it will be useful to have some special vocabulary for them.

It is standard to speak of truth and falsity together as ***truth values*** and to abbreviate their names as **T** and **F**, respectively. So, to say that an argument is valid is to claim that the pattern of truth values for its premises and conclusion shown in Figure 1.2.1-1 is impossible. That is (using some of the other terminology we have available), a conclusion is entailed by a set of assumptions when it is impossible for the truth value of the conclusion to be **F** when each of the assumptions has the truth value **T**.

$$\begin{array}{rc} premise & \mathbf{T} \\ premise & \mathbf{T} \\ \vdots & \vdots \\ premise & \mathbf{T} \\ \hline conclusion & \mathbf{F} \end{array}$$

Fig. 1.2.1-1. The pattern of truth values that is impossible when an argument is valid.

Since to speak of a risk of error is to speak of a possibility of error, it is also useful to have some vocabulary for speaking of possibility and impossibility. The sort of impossibility in question when we speak of entailment is a very strong sort that may be referred to as ***logical impossibility***. A description of a situation that runs counter to the laws of physics (for example, a locomotive floating 10 feet above the earth's surface without any abnormal forces acting on it) might be said to be physically impossible; but it need not be logically impossible, and we must consider many physical impossibilities when deciding whether a conclusion is deductively valid. Knowledge that someone was in Boston at 10:00 A.M. EST on January 1, 1980 renders the conclusion that she was not in New York at 10:01 A.M. EST that same day unassailable

from the point of view of common sense. But this is not a deductively valid conclusion, for presence in those places at those times is logically possible and it would remain so if the difference in time was made so small as to imply travel faster than the speed of light.

We can say that something is impossible by saying that "there is no possibility" of it being true. In saying this, we use a form of words analogous to one we might use to say that there is no photograph of Abraham Lincoln chopping wood. That is, in saying "there is no possibility," we speak of possibilities as if they were things like photographs. This way of speaking about possibilities is convenient, so it is worth spending a moment thinking about what sort of things possibilities might be. The sort of possibility of chief interest to us is a complete state of affairs or state of the world, where this is understood to include facts concerning the full course of history, both past and future. Since Leibniz, philosophers have used the phrase **possible world** as a particularly graphic way of referring to possibilities in this sense. For instance, Leibniz held that the goodness of God implied that the actual world must be the best of all possible worlds, and by this he meant that God made the entire course of history as good as it was logically possible for it to be.

The term *world* could be misleading since it might suggest something that a physicist would describe as a system that can have many different states; in our usage, however, a possible world is less like a system than a particular one of its states or a particular history tracing these states through time; and these are not states of one among many systems but of a global system including everything there is. That is, the phrase *possible world* could be restated more accurately, though less conveniently, using the phrase *possible history of the universe*.

To say that something is logically *im*possible is to say that it is true in *no* logically possible world. The weaker claim of physical impossibility says only that a claim is found to be true nowhere in the narrower range of physical possibilities or physically possible worlds. But what sets the limits of the particular range of possible worlds considered in deductive logic? If we move beyond the bounds of physical laws, what is left to stop us from saying anything? I claimed earlier that travel faster than the speed of

light was logically possible. Is the same true for presence in two places at the same time or travel backward in time? We will not need specific answers to such questions, but we should have some general sense of the basis for answering questions like them.

Unfortunately, there is no uncontroversial view of this. However, the issues at dispute between the alternatives are largely independent of the content of deductive logic, so we will consider only one respectable account without giving equal attention to its rivals. The association between logic and language mentioned in 1.1 provides this account. It traces the norms of deductive inference to rules governing the meanings of sentences, and these rules might be held to set the limits of the range of logical possibilities. So one way to answer the question of whether a claim of impossibility is a claim of *logical* impossibility is ask whether the description of what is said to be impossible is really disallowed by the semantic rules of the language in which it is stated. To decide whether being in two places at the same time is a logical impossibility is to decide whether a sentence like *I was in two places at the same time* is even a meaningful description. To say that being in two places at the same time was logically impossible would not be to describe such a situation and go on to say that it is impossible but instead to claim that no such situation could even be coherently described.