1.1.4. Entailment

To say that our reasoning is risk-free when we confine ourselves to the extraction of information is to deny the possibility of going wrong when the premises and conclusion of an argument are related in this way. In this sort of case, we will say that the conclusion is **entailed by** the premises. So the extraction of information is characterized by a relation of **entailment** between the initial data and the information extracted from it. If we speak in terms of arguments, entailment is a relation that may or may not hold between given premises and a conclusion, and we can speak of an argument as having the property of **validity** if its premises do entail its conclusion. We will say also that the conclusion of an argument with this property is a **valid conclusion** from its premises. Figure 1.1.4-1 summarizes these ways of stating the relation of entailment between a set of premises or assumptions Γ and a conclusion φ.

> the assumptions Γ **entail** the conclusion φ
> the conclusion φ **is entailed by** the assumptions Γ
> the conclusion φ **is a valid conclusion from** the assumptions Γ
> the argument Γ / φ **is valid**

Fig. 1.1.4-1. Several ways of stating a relation of entailment.

We will use the sign ⇒ (**rightwards double arrow**) as shorthand for the verb *entails*, so we add to the English expressions in Figure 1.1.4-1 the symbolic expression Γ ⇒ φ as a way of saying that the premises Γ entail the conclusion φ. Using this sign, we can express the validity of argument in Figure 1.1.2-1 by writing

All humans are mortal, *Socrates is human* ⇒ *Socrates is mortal*

Notice that a symbolic expression of the form Γ / φ (which amounts to the English expression *the argument formed of premises Γ and conclusion φ*) is a noun phrase and is comparable in this respect to the expression $x + y$ (which amounts to the English *the sum of x and y*) while an expression of the form Γ ⇒ φ is a sentence (and is comparable to the expression $x < y$).

Entailment and validity are normative concepts since they apply to arguments that are *good* in a certain respect. They might be said to concern the relation that *ought* to hold between the premises and conclusion of an inference if it purports to be risk-free. But

this is not to say that all inferences ought to be deductive, that all arguments ought to be valid. There are some contexts, such as mathematical proof, where the level of security provided by deductive reasoning is required. Still, it often cannot be expected and, in many cases, it would be undesirable. We saw in 1.1.3 that, when our aim is to generalize or explain, deductive inference is not what we want in the end (though it may help along the way).