

Phi 270 F99 test 5 in pdf format

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer. Notice the additional instructions given for each of the first two.

1. *Sam mentioned someone Tina didn't know.* [Give this analysis also using an unrestricted quantifier.]
[answer]
2. *Every shoe fit someone.* [This sentence is ambiguous. Analyze it in two different ways, and describe a situation in which the sentence is true on one of your interpretations and false on the other.]
[answer]
3. *Sam found at least two pieces.*
[answer]

Analyze the sentence below using each of the two ways of analyzing definite descriptions. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

4. *The elephant standing on Sam sighed.*
[answer]

[The following question was on a topic not covered in FO4] Put the following sentence into prenex normal form (i.e., into a form which contains no restricted quantifiers and in which no quantifier is in the scope of a connective). Show each step where you move a quantifier past a connective separately.

5. $\neg (\forall x: Px \wedge \exists y: Rxy) \exists z: Sxz$
[answer]

Use derivations to show that the following argument is valid. You may use attachment rules (but not replacement by equivalence).

6.
$$\frac{\forall x (\forall y: Rxy) (Ryx \rightarrow Rxx) \quad \exists x (\exists y: Rxy) Ryx}{\exists x Fxx}$$

That is: *Some finding is different from something* \Rightarrow *Something is such that something different from it is a finding* [but don't hesitate to ignore the English if it doesn't help].

[answer]

Use a derivation to show that the following argument is not valid and describe a structure dividing an open gap.

7.
$$\frac{\exists x Fx \quad (\exists x: Gx) Hx}{\exists x (Fx \wedge Hx)}$$

[answer]

Complete the following to give a definition of entailment by a single sentence (i.e., implication) in terms of truth values and possible worlds:

8. A sentence ϕ entails a sentence ψ if and only if ...
[answer]

Complete the following truth table by calculating the truth value of the

sentence on the given assignment. Show the value of each component by writing it under the main connective of that component.

$$9. \frac{A \ B \ C \ D}{T \ F \ F \ T} \neg (A \wedge B) \rightarrow (C \vee \neg D)$$

[answer]

Describe a structure (i.e., an assignment of extensions to the non-logical vocabulary) which makes the sentences below all true. (You may use either tables or a diagram.)

$$10. a = fb, fb = fc, fa = c, Pa, Pb, \neg Pc, Rab, Rbc, Rc(fb)$$

[answer]

Phi 270 F99 test 5 answers

1. *Sam mentioned someone Tina didn't know*
someone Tina didn't know is such that (Sam mentioned him or her)

$(\exists x: x \text{ is a person Tina didn't know}) \text{ Sam mentioned } x$

$(\exists x: x \text{ is a person} \wedge \neg \text{Tina knew } x) \text{ Sam mentioned } x$

$(\exists x: Px \wedge \neg Ktx) Msx$

$\exists x ((Px \wedge \neg Ktx) \wedge Msx)$

[K: $\lambda xy (x \text{ knew } y)$; M: $\lambda xy (x \text{ mentioned } y)$; P: $\lambda x (x \text{ is a person})$; s:

Sam; t: *Tina*]

2. *first analysis:*

Every shoe fit someone

every shoe is such that (it fit someone)

$(\forall x: x \text{ is a shoe}) x \text{ fit someone}$

$(\forall x: Sx) \text{ someone is such that } (x \text{ fit him or her})$

$(\forall x: Sx) (\exists y: y \text{ is a person}) x \text{ fit } y$

$(\forall x: Sx) (\exists y: Py) Fxy$

second analysis:

Every shoe fit someone

someone is such that (every shoe fit him or her)

$(\exists x: x \text{ is a person}) \text{ every shoe fit } x$

$(\exists x: Px) \text{ every shoe is such that (it fit } x)$

$(\exists x: Px) (\forall y: y \text{ is a shoe}) y \text{ fit } x$

$(\exists x: Px) (\forall y: Sy) Fyx$

[F: $\lambda xy (x \text{ fit } y)$; P: $\lambda x (x \text{ is a person})$; S: $\lambda x (x \text{ is a shoe})$]

The first is true and the second false if every shoe could be worn but not all by the same person

3. *Sam found at least two pieces*

at least two pieces are such that (Sam found them)

$(\exists x: x \text{ is a piece}) (\exists y: y \text{ is a piece} \wedge \neg y = x) (\text{Sam found } x \wedge \text{Sam found } y)$

$(\exists x: Px) (\exists y: Py \wedge \neg y = x) (Fsx \wedge Fsy)$

[F: $\lambda xy (x \text{ found } y)$; P: $\lambda x (x \text{ is a piece})$; s: *Sam*]

4. using Russell's analysis:

The elephant standing on Sam sighed

The elephant standing on Sam is such that (it sighed)

$(\exists x: x \text{ and only } x \text{ is an elephant standing on Sam}) x \text{ sighed}$

$(\exists x: x \text{ is an elephant standing on Sam} \wedge (\forall y: \neg y = x) \neg y \text{ is an elephant standing on Sam}) Sx$

$(\exists x: (x \text{ is an elephant} \wedge x \text{ is standing on Sam}) \wedge (\forall y: \neg y = x) \neg (y \text{ is an elephant} \wedge y \text{ is standing on Sam})) Sx$

$(\exists x: (Ex \wedge Txs) \wedge (\forall y: \neg y = x) \neg (Ey \wedge Tys)) Sx$

or:

$(\exists x: (Ex \wedge Txs) \wedge (\forall y: Ey \wedge Tys) x = y) Sx$

using the description operator:

The elephant standing on Sam *sighed*

S (*the elephant standing on Sam*)

S (ιx *x is an elephant standing on Sam*)

S (ιx (*x is an elephant* \wedge *x is standing on Sam*)))

S(ιx ($Ex \wedge Txs$)))

[E: λx (*x is an elephant*); S: λx (*x sighed*); T: λxy (*x is standing on y*); s: *Sam*]

5. [The following question was on a topic not covered in FO4]

$\neg (\forall x: Px \wedge \exists y Rxy) \exists z Sxz$

$\neg \forall x ((Px \wedge \exists y Rxy) \rightarrow \exists z Sxz)$

$\exists x \neg ((Px \wedge \exists y Rxy) \rightarrow \exists z Sxz)$

$\exists x \neg (\exists y (Px \wedge Rxy) \rightarrow \exists z Sxz)$

$\exists x \neg \forall y ((Px \wedge Rxy) \rightarrow \exists z Sxz)$

$\exists x \exists y \neg ((Px \wedge Rxy) \rightarrow \exists z Sxz)$

$\exists x \exists y \neg \exists z ((Px \wedge Rxy) \rightarrow Sxz)$

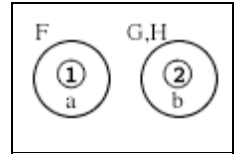
$\exists x \exists y \forall z \neg ((Px \wedge Rxy) \rightarrow Sxz)$

6.

	$\forall x (\forall y: Rxy) (Ryx \rightarrow Rxx)$	a:3
	$\exists x (\exists y: Rxy) Ryx$	1
	(a) $(\exists y: Ray) Rya$	2
	(b) Rab	(4)
	Rba	(5)
	(3 UI) $(\forall y: Ray) (Rya \rightarrow Raa)$	b:4
	(4 SB) $Rba \rightarrow Raa$	5
	(5 MPP) Raa	(6)
	(6 EG) $\exists x Rxx$	X, (7)
	•	
	(7 QED) $\exists x Rxx$	2
	(2 PRCh) $\exists x Rxx$	1
	(1 PCh) $\exists x Rxx$	

7.

	$\exists x Fx$		1
	$(\exists x: Gx) Hx$		2
	(a) Fa		(6)
	(b) Gb Hb		(7)
	$\forall x \neg (Fx \wedge Hx)$	a:4,b:5	
4 UI	$\neg (Fa \wedge Ha)$		6
5 UI	$\neg (Fb \wedge Hb)$		7
6 MPT	$\neg Ha$		
7 MPT	$\neg Fb$		
	○		$Fa, Gb, Hb, \neg Ha, \neg Fb \Rightarrow \perp$
	\perp		3
3 NcP	$\exists x (Fx \wedge Hx)$		2
2 PRCh	$\exists x (Fx \wedge Hx)$		1
1 PCh	$\exists x (Fx \wedge Hx)$		



8. A sentence ϕ entails a sentence ψ if and only if there is no possible world in which ϕ is true but ψ is false (or: if and only if ψ is true in every possible world in which ϕ is true)

9.

A	B	C	D		$\neg (A \wedge B) \rightarrow (C \vee \neg D)$
T	F	F	T		T F \textcircled{F} F F

10. $a = fb, fb = fc, fa = c, Pa, Pb, \neg Pc, Rab, Rbc, Rc(fb)$

<i>alias sets</i>	<i>IDs</i>	<i>values</i>	<i>resources</i>	<i>values</i>
a	1	a: 1	Pa	P1: T
fb		f2: 1	Pb	P2: T
fc		f3: 1	$\neg Pc$	P3: F
<hr/>				
b	2	b: 2	Rab	R12: T
<hr/>				
c	3	c: 3	Rbc	R23: T
fa		f1: 3	Rc(fb)	R31: T

range: 1, 2, 3

a	b	c	τ	f τ	τ	P τ	R	1	2	3
1	2	3	1	3	1	T	1	F	T	F
			2	1	2	T	2	F	F	T
			3	1	3	F	3	T	F	T

