

**Phi 270 F02 test 4** in pdf format

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *Notice the special instructions for 2.*

1. *Only bears performed.*

[ answer ]

2. *If everyone cheered, the elephant bowed.* [In this case, restate your answer using an unrestricted quantifier.]

[ answer ]

3. *No one laughed at any performers except clowns.*

[ answer ]

Synthesize an English sentence with the following logical form:

4.  $(\forall x: Px \wedge Cxt) Ctx$

[C:  $\lambda xy (x \text{ called } y)$ ; P:  $\lambda x (x \text{ is a person})$ ; t: *Tom*]

[ answer ]

Use derivations to establish the validity of the following arguments. You may use attachment rules.

5.  $\forall x Fx$

$\forall x \neg (Fx \wedge Gx)$

$\forall x \neg Gx$

[ answer ]

6.  $\forall x (\forall y: Fy) Rxy$

$(\forall x: Fx) \forall y Ryx$

[ answer ]

Use a derivation to show that the following argument is not valid and describe a structure (by using either a diagram or tables) that divides one of the derivation's open gaps.

7.  $\forall x Rax$

$(\forall x: Rbx) \neg Rxa$

$\forall x \neg Rbx$

[ answer ]

**Phi 270 F02 test 4 answers**

1. *Only bears performed*

$(\forall x: \neg x \text{ is a bear}) \neg x \text{ performed}$

$(\forall x: \neg Bx) \neg Px$

[B:  $\lambda x (x \text{ is a bear})$ ; P:  $\lambda x (x \text{ performed})$ ]

2. *If everyone cheered, the elephant bowed everyone cheered  $\rightarrow$  the elephant bowed*

$(\forall x: x \text{ is a person}) x \text{ cheered} \rightarrow \text{the elephant bowed}$

$(\forall x: Px) Cx \rightarrow Be$

$\forall x (Px \rightarrow Cx) \rightarrow Be$

[B:  $x \text{ bowed}$ ; C:  $x \text{ cheered}$ ; P:  $x \text{ is a person}$ ; e: *the elephant*]

*Incorrect:*

$(\forall x: Px) (Cx \rightarrow Be)$  or:  $\forall x (Px \rightarrow (Cx \rightarrow Be))$

these say: *If anyone cheered, the elephant bowed*

3. *No one laughed at any performers except clowns*  
*all performers except clowns are such that (no one laughed at them)*

$(\forall x: x \text{ is a performer} \wedge \neg x \text{ is a clown})$  no one laughed at x  
 $(\forall x: x \text{ is a performer} \wedge \neg x \text{ is a clown}) (\forall y: y \text{ is a person}) \neg y$   
 laughed at x

$(\forall x: Fx \wedge \neg Cx) (\forall y: Py) \neg Lyx$

[C:  $\lambda x (x \text{ is a clown})$ ; F:  $\lambda x (x \text{ is a performer})$ ; P:  $\lambda x (x \text{ is a person})$ ;

L:  $\lambda xy (x \text{ laughed at } y)$ ]

Incorrect:

$(\forall y: Py) \neg (\forall x: Fx \wedge \neg Cx) Lyx$

says: *No one laughed at all performers who weren't clowns*

4.  $(\forall x: x \text{ is a person} \wedge x \text{ called Tom})$  Tom called x  
 $(\forall x: x \text{ is a person who called Tom})$  Tom called x  
*everyone who called Tom is such that (Tom called him or her)*  
*Tom called everyone who called him*

5.

$\forall x Fx$	$a:2$
$\forall x \neg (Fx \wedge Gx)$	$a:3$
(a)	
2 UI	$Fa$ (4)
3 UI	$\neg (Fa \wedge Ga)$ 4
4 MPT	$\neg Ga$ (5)
	•
5 QED	$\neg Ga$ 1
1 UG	$\forall x \neg Gx$

6.

$\forall x (\forall y: Fy) Rxy$	$b:3$
(a)	
	$Fa$ (4)
3 UI	$(\forall y: Fy) Rby$ a:4
4 SB	$Rba$ (5)
	•
5 QED	$Rba$ 2
2 UG	$\forall y Rya$ 1
1 RUG	$(\forall x: Fx) \forall y Ryx$

7.

	$\forall x Rax$	$a:3, b:4, c:5$
	$(\forall x: Rbx) \neg Rxa$	$c:6, a:7, b:8$
	Ⓢ	
		Rbc (6)
		Raa (7)
3 UI		Rab
4 UI		Rac
5 UI		$\neg Rca$
6 SB		$\neg Rba$
7 SC		
		$\neg Rbb$
		○ Rbc, Raa, Rab, Rac, $\neg Rca$ , $\neg Rba$ , $\neg Rbb \Rightarrow \perp$
		⊥ 9
9 IP		Rbb 8
		$\neg Rba$
		○ Rbc, Raa, Rab, Rac, $\neg Rca$ , $\neg Rba \Rightarrow \perp$
		⊥ 8
8 MCR		⊥ 2
2 RAA		$\neg Rbc$ 1
1 UG	$\forall x \neg Rbx$	

Counterexample presented by tables

range: 1, 2, 3	<table style="border-collapse: collapse; text-align: center;"> <tr><td>a</td><td>b</td><td>c</td></tr> <tr><td>1</td><td>2</td><td>3</td></tr> </table>	a	b	c	1	2	3	<table style="border-collapse: collapse; text-align: center;"> <tr><td>R</td><td>1</td><td>2</td><td>3</td></tr> <tr><td>1</td><td>T</td><td>T</td><td>T</td></tr> <tr><td>2</td><td>F</td><td>F</td><td>T</td></tr> <tr><td>3</td><td>F</td><td>F</td><td>F</td></tr> </table>	R	1	2	3	1	T	T	T	2	F	F	T	3	F	F	F
a	b	c																						
1	2	3																						
R	1	2	3																					
1	T	T	T																					
2	F	F	T																					
3	F	F	F																					

Grayed values are not required to divide either gap;  
 the value for R22 is not required to divide the 2nd gap

Counterexample presented by a diagram

