7.7.s. Summary

7.7.1. Our system of derivations generalizations does not answer all questions concerning the validity of arguments; indeed, it has been shown that no system can answer all such questions (if its answers are all correct). However, our system is sound and complete. That is, it declares valid only arguments that are valid, and it does affirm the validity of all valid arguments. These properties make up more than half of what we might like a system to do: a sound and complete system always gives a correct answer concerning valid arguments and never gives an incorrect answer concerning arguments that are not valid (though it may give no answer at all in the case of such arguments). We can still establish the soundness of our current system much as before, and we can establish completeness by showing (i) that any derivation that does not close will contain a path that is fully developing (in the sense that every way of developing it is employed at some point) and (ii) that any fully developing path is divided by some interpretation. To show (i) is to show that a system is thorough, and to show (ii) is to show that it is effectual.

7.7.2. We must refine our notion of interpretation to recognize the possibility that the non-logical vocabulary of a derivation may increase as it develops, and we need to modify the definition of soundness, too. The rules for universals may introduce terms, and a structure dividing a gap to which these rules are applied may assign inconvenient values, or no values at all, to these terms. So we will ask for soundness only that we be able to find a structure dividing a child gap that agrees with the old structure on the vocabulary appearing before the rule was applied. This new definition of utter and minimally sound rules still implies the soundness of our system.

7.7.3. A derivation may develop forever due to continual input of new terms for which universals are exploitable as we plan for recurrent universal goals or instantiate universals containing functors. To establish thoroughness, we must insure that all approaches to closing the gap are explored in the course of this development. We can do this by imposing an order of procedure that rations the terms used to instantiate over the course of time, requiring a full cycle in the application of other rules before new terms are introduced. While this rule insures thoroughness, it makes more sense in practice simply to take on the responsibility for being thorough.

7.7.4. Infinite derivations are not static structures but growing lines of development. This leads to changes in the way we argue for the existence of structures dividing paths that never close off. We collect the active resources and goals that appear in the course of a gap's development as accumulated resources and accumulated goals distinguishing as ultimate those resources that are never exploited. When a gap is fully developing, its ultimate resources are limited to atomic sentences and their negations. We can show that any fully developing gap leads us to a structure that makes its accumulated resources true and its accumulated goals false. Although there are thus enough structures to meet our needs, some of the flexibility we have had in choosing structures is now gone: we can no longer expect to add values freely to the range of a structure since some sentences are true only when the referential range has a limited size.

7.7.x. Exercise questions

Use derivations to check each of the claims below; if a derivation indicates that a claim fails, describe a structure that divides an open gap.

1. Fa ⇒ ∀x Fx

2. ∀x Rxx ⇒ ∀x Rxa

3. ∀x ¬ Fx ⇔ ¬ ∀x Fx

4. *No widget is a gadget*, *No gizmo is a widget* ⇒ *No gizmo is a gadget*

5. *No widget is a gadget* ⇔ *Not every widget is a gadget*

6. *Everything is either finished or unstarted* ⇒ *Either everything is finished or everything is unstarted*

7. ¬ ∀x ¬ ∀y Rxy ⇒ ∀x ¬ ∀y ¬ Rxy

Homework assigned Mon 11/14 and due Wed 11/16

(i) Use a derivation to check the following and give a structure dividing an open gap:

(∀x: Fx ∧ Gx) Hx, Fa ∧ Ga / (∀x: Fx) (Gx ∧ Hx)

[Remember that you have not reached a dead end until you have exploited every generalization for each term in the gap]

(ii) Analyze: *No one who attempts everything finishes anything*