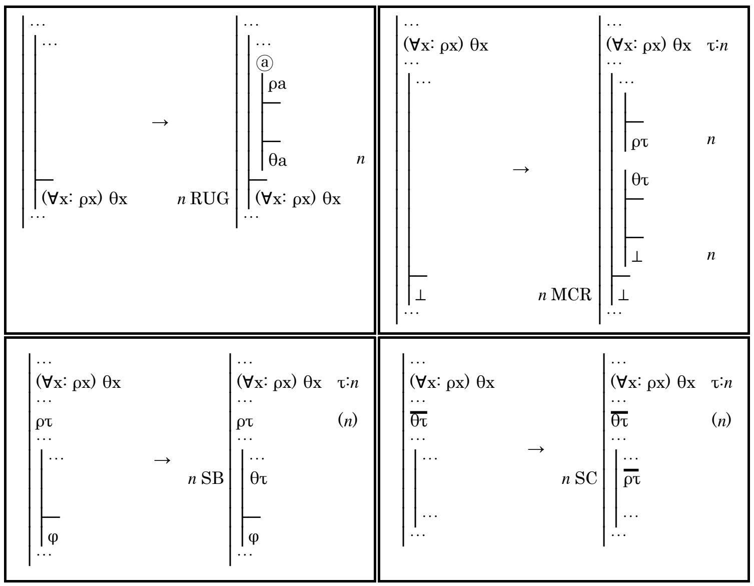
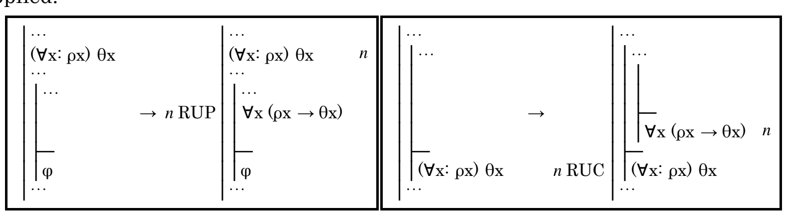
7.6.s. Summary

7.6.1. While the logical properties of an unrestricted universal may be tied to those of its instances, in the case of a restricted universal, we use the idea of a conditioned instance, an application of the quantified predicate to a term made conditional on an application of the restricting predicate to the same term. Laws for restricted universals then combine the ideas used to capture the role of unrestricted universals with ideas used to capture the role of conditionals. The law for the restricted universal as a conclusion requires that we conclude an instance for a parameter but allows us to add a supposition that applies the restricting predicate of the universal to the parameter. For restricted universal resources, we have two detachment arguments, singular *Barbara* and singular *Camestres*, named after similar syllogistic patterns. These principles are analogous to uses of *modus ponens* and *modus tollens* for a conditioned instance of the universal and the first amounts to a sort of restricted universal instantiation. There is also a principle for the restricted universal as a premise in *reductio* arguments that again reflects the role of its conditioned instances.

7.6.2. The most characteristic derivation rules for restricted universals — Restricted Universal Generalization (RUG), Singular Barbara (SB), Singular Camestres (SC), and Making a Counterexample for *Reductio* (MCR) — are rules that implement these principles.



But it is also possible to capture the properties of restricted universals by rules, Restricted Universal Premise (RUP) and Restricted Universal Conclusion (RUC), that support restating restricted universal resources and goals using the unrestricted universal so that rules for that quantifier and conditionals may be applied.



7.6.x. Exercise questions

1. Use the system of derivations to establish each of the following. You may use detachment and attachment rules. Exercises **h** and **i** are contrived to require the rule MCR for exploiting restricted universals; any restricted universals in the others may be exploited using the detachment rules.

- **a.** $\forall x\, \forall y\, Rxy,\ (\forall x{:}\ Rxx)\, Gx \Rightarrow Ga$
- **b.** $(\forall x{:}\ Fx)\, Gx \Leftrightarrow \forall x\, (Fx \to Gx)$
- **c.** $Fa \Leftrightarrow (\forall x{:}\ x = a)\, Fx$
- **d.** $\forall x\, \forall y\, (Rxy \to \neg Ryx) \Rightarrow \forall x\, (\forall y{:}\ \neg\, x = y)\, \neg\, (Rxy \land Ryx)$
- **e.** $\forall x\, (\forall y{:}\ \neg\, x = y)\, \neg\, (Rxy \land Ryx),\ \forall x\, \neg\, Rxx \Rightarrow \forall x\, \forall y\, (Rxy \to \neg Ryx)$
- **f.** *Everyone loves everyone who loves anyone* ⇒ *If anyone loves anyone, then everyone loves everyone*
- **g.** $\forall x\, (\forall y{:}\ gx = y)\, Fy \Rightarrow \forall x\, F(g(hx))$
- **h.** $\forall x\, \forall y\, Rxy,\ (\forall x{:}\ \forall y\, Ryx)\, (Fx \to Gx) \Rightarrow (\forall x{:}\ Fx)\, Gx$
- **i.** *Al said everything he remembered*, *Al is a person who said nothing*, *Anyone who remembered nothing forgot everything* ⇒ *Al forgot everything*

2. In the following, certain alternative expressions are enclosed in brackets and separated by vertical bars. Choose one of each alternative pair of premises and one of each alternative pair of words or phrases in the conclusion so as to make a valid argument; then analyze the premises and conclusion and construct a derivation to show that the argument is valid. You may use detachment and attachment rules.

a. *Every road sign was colored*
[*Every stop sign was a road sign* | *Every road sign was a traffic marker*]
[*If anything was red, it was colored* | *If anything was colored, it was painted*]
Every [*stop sign* | *traffic marker*] *was* [*red* | *painted*]

b. *No road sign was colored*
[*Every stop sign was a road sign* | *Every road sign was a traffic marker*]
[*If anything was red, it was colored* | *If anything was colored, it was painted*]
No [*stop sign* | *traffic marker*] *was* [*red* | *painted*]

c. *Only road signs were colored*
[*Every stop sign was a road sign* | *Every road sign was a traffic marker*]
[*If anything was red, it was colored* | *If anything was colored, it was painted*]
Only [*stop signs* | *traffic markers*] *were* [*red* | *painted*]

d. *Among road signs all except colored ones were replaced*
[*Every stop sign was a road sign* | *Every road sign was a traffic marker*]
[*If anything was red, it was colored* | *If anything was colored, it was painted*]
Among [*stop signs* | *traffic markers*] *all except* [*red* | *painted*] *ones were replaced*

e. *Everyone watched every snake*
[*Every cobra is a snake* | *Every snake is a reptile*]
Everyone watched every [*cobra* | *reptile*]

f. *No one watched every snake*
[*Every cobra is a snake* | *Every snake is a reptile*]
No one watched every [*cobra* | *reptile*]

g. *No one watched any snake*
[*Every cobra is a snake* | *Every snake is a reptile*]
No one watched any [*cobra* | *reptile*]

h. *Everyone who likes every snake was pleased*
[*Every cobra is a snake* | *Every snake is a reptile*]
Everyone who likes every [*cobra* | *reptile*] *was pleased*

i. *Everyone who likes a snake was pleased*
[*Every cobra is a snake* | *Every snake is a reptile*]
Everyone who likes a [*cobra* | *reptile*] *was pleased*

Homework assigned Fri 11/11 and due Mon 11/14

Use derivations to show:

$$\forall x\, (\forall y{:}\ Py \land \neg Fy)\, \neg Nxy \Rightarrow \forall y\, (\forall x{:}\ Px)\, (Nyx \to Fx)$$