6.4.s. Summary

6.4.1. Logical forms (without free variables) may be given semantic values by assigning values to the non-logical vocabulary they contain; that is, they can be given extensions (or intensions) by an extensional (or intensional) *interpretation* of this vocabulary. The extensions of predicates and functors are functions that take as input reference values from a referential range **R** that must be specified along with an extensional interpretation; the range and the interpretations of non-logical vocabulary together constitute a structure for any expressions formed using only the non-logical vocabulary that is interpreted in the structure. We assume each value of the range is labeled by an ID.

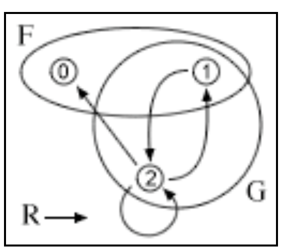
6.4.2. A referential range may be depicted by points in a plane labeled by their IDs, and further labeling and other devices can depict extensions of non-logical vocabulary on this range. Terms may be used to label the points that represent their reference values, and one-place predicates may label the points they are true of. Alternatively a one-place predicate may label a line enclosing the set of all points it is true of; this set is one way of representing its extension. If the extension of a predicate of more than one place is thought of as a set, it must be a set of ordered pairs, triples, or other *n-tuples*; these may be represented by arrows (perhaps with legs) that indicate the order of values in the *n*-tuple. The reference functions that are extensions of functors are not easily depicted in this way, but they may be displayed in tables analogous to mathematical tables. We may calculate the extensions that structures give to expressions by using a table analogous to a truth table, with all the information in a structure providing the basis for the calculation of a single row.

6.4.3. Structures are now the appropriate counterexamples to claims of validity. To build a structure that divides a dead-end gap, we take the alias sets of the gap and choose a range that contains a value corresponding to each alias set. Then we assign extensions to unanalyzed terms and functors so that the reference value each compound term will be the value corresponding to the term's alias set. Finally, we assign extensions to predicates by seeing what terms the resources affirm or deny these predicates of. Our new rules for closing gaps ensure that these instructions are consistent and that a structure built in this way will divide the dead-end gap. Such a structure can also be found as at least a part of a possible world.

6.4.x. Exercise questions

1. Each of **a**, **b**, and **c** gives a structure in one of the two sorts of presentation described in this section—by a diagram or by tables. Present each of them in the other way.

a.



b.

τ	Fτ
0	T
1	T
2	F

τ	Gτ
0	F
1	F
2	T

R	0	1	2
0	T	T	T
1	F	T	F
2	F	T	T

c.

τ	Fτ
0	T
1	T
2	F

τ	Gτ
0	F
1	T
2	T

τ	Hτ
0	T
1	F
2	T

R	0	1	2
0	F	T	F
1	T	F	F
2	F	T	F

2. Calculate a truth value for each of the following sentences on the structure used as the chief example in this section (see, for example, Figure 6.4.2-7):

a. (Fa ∨ Gb) → Rab

b. R(fca)(fac)

c. fab = fba

3. Use derivations to check each of the claims below; if a claim of entailment fails, use either tables or a diagram to present a structure that divides an open gap.

a. a = a → Fa ⇒ Fa

b. ¬ (Fa ∧ Fb) ⇒ ¬ Fa → ¬ Fb

c. a = b ∨ b = a ⇒ a = b ∧ b = a

d. Fa → a = b, ga = b, Ra(ga) → Fa, F(ga) ⇒ Raa → R(ga)(ga)

e. a = b → Rac, ¬ a = b → Rbc ⇒ Rbc

Topics for test 3

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

- *Analysis*. Two sorts of questions are possible here corresponding to the sorts of analyses you have done in chs. 5 and 6: (i) analysis by truth-functional connnectives alone, with atomic sentences as the ultimate components (the emphasis will, of course, be on conditionals—i.e., on the symbolic representation of *if*, *only if*, and *unless*) and (ii) analysis using not only truth-functional connnectives but also predicates, individual terms, and functors.
- *Derivations*. Be able to construct derivations to show that entailments hold and to show that they fail. I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. There may be some derivations where detachment and attachment rules may be used and where they will shorten the proof. But there will be others where you must rely on other rules, either because detachment and attachment rules do not apply or because I tell you not to use them. In particular, be ready to use the rule RC (Rejecting a Conditional) from ch. 5.

  Remember that, if a derivation involves predicates and functors, presenting a counterexample will require the description of a structure and not merely an assignment of truth values. You will be allowed to use either tables or diagrams to describe structures.