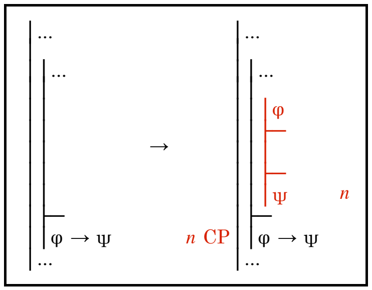
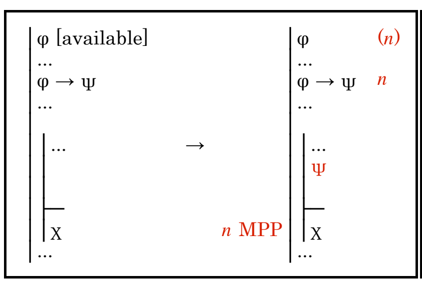
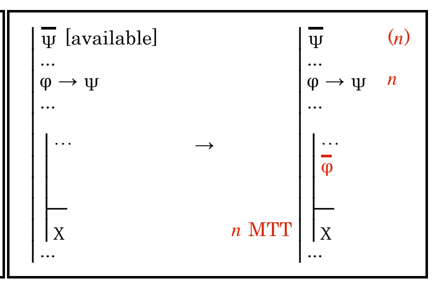
5.3.s. Summary

5.3.1. The truth conditions of the conditional recall the definition of implication. Indeed, an implication $\varphi \Rightarrow \psi$ will hold if and only if the conditional $\varphi \rightarrow \psi$ is a tautology. We can apply similar ideas to conditionals that are conclusions from factual premises by considering a notion of relative implication, implication depending on factual information. This idea appears in our law for the conditional as a conclusion. An entailment $\Gamma \Rightarrow \varphi \rightarrow \psi$ holds when $\Gamma, \varphi \Rightarrow \psi$—i.e., when ψ is implied by φ given the further premises Γ. The first of these entailments is a conditionalization of the second, and the second asserts the validity of a hypothetical argument. So an argument with a conditional conclusion is valid if and only if the hypothetical argument it conditionalizes is also valid. The derivation rule implementing this idea is Conditional Proof (CP).



5.3.2. The detachment principles for the conditional include the well-known *modus ponendo ponens* (usually called *modus ponens*), which is implemented as a rule Modus Ponendo Ponens (MPP), and a second detachment principle *modus tollendo tollens* (usually called *modus tollens*), which is implemented as a rule Modus Tollendo Tollens (MTT). *Modus ponens* in particular can be understood as the use of a conditional as an inference ticket licensing transitions from its antecedent to its consequent.





5.3.x. Exercise questions

1. Use derivations to establish each of the following. Notice that several are claims of equivalence and require two derivations. All these derivations are designed for the use of detachment rules (especially MPP and MTT), and a number will be quite long if they are not used. Attachment rules from previous chapters will occasionally be useful, and (since we do not yet have a full set of rules for the conditional) they are required in one of the derivations for **k**. Finally, note the leftwards arrow in the second premise of **b**. Although rules like MPP are written using a rightwards arrow they also apply to conditionals written using a leftwards arrow since a conditional $\psi \leftarrow \varphi$ is just an alternative way of writing $\varphi \rightarrow \psi$ and plays the same role in derivations.

   a. $B \rightarrow C, A \rightarrow B \Rightarrow A \rightarrow C$

   b. $A \rightarrow B, C \leftarrow B, C \rightarrow D \Rightarrow A \rightarrow D$

   c. $A \rightarrow (B \rightarrow C) \Rightarrow (A \rightarrow B) \rightarrow (A \rightarrow C)$

   d. $A \rightarrow (B \rightarrow C), A \rightarrow \neg C \Rightarrow B \rightarrow \neg A$

   e. $\neg A \Leftrightarrow A \rightarrow \neg A$

   f. $A \rightarrow B \Leftrightarrow \neg B \rightarrow \neg A$

   g. $A \rightarrow B \Leftrightarrow \neg (A \wedge \neg B)$

   h. $A \rightarrow (B \rightarrow C) \Leftrightarrow (A \wedge B) \rightarrow C$

   i. $(A \rightarrow B) \wedge (A \rightarrow C) \Leftrightarrow A \rightarrow (B \wedge C)$

   j. $(A \rightarrow C) \wedge (B \rightarrow C) \Leftrightarrow (A \vee B) \rightarrow C$

   k. $(A \rightarrow B) \wedge (B \rightarrow C) \Leftrightarrow (A \vee B) \rightarrow (B \wedge C)$

2. Give English sentences illustrating **d**, **f**, **g**, and **k** of **1**. (Notice that **k** tells how to restate a particular sort of conjunction of conditionals, one that might be called a ***linked conditional***.)

Homework assigned Wed 10/12 and due Mon 10/17

Use derivations to show the following:

$$A \rightarrow (B \rightarrow C), (A \wedge D) \rightarrow B \Rightarrow \neg C \rightarrow (A \rightarrow \neg D)$$

(Hint: you will need to use detachment rules from both 5.3 and 4.3.)