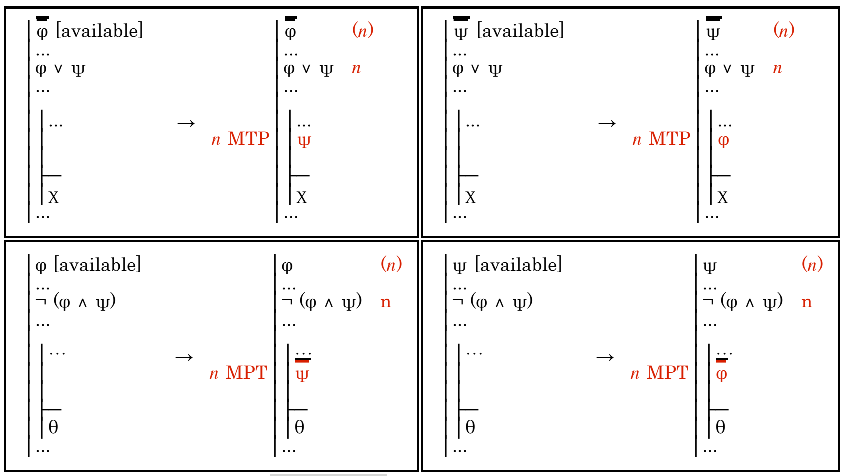
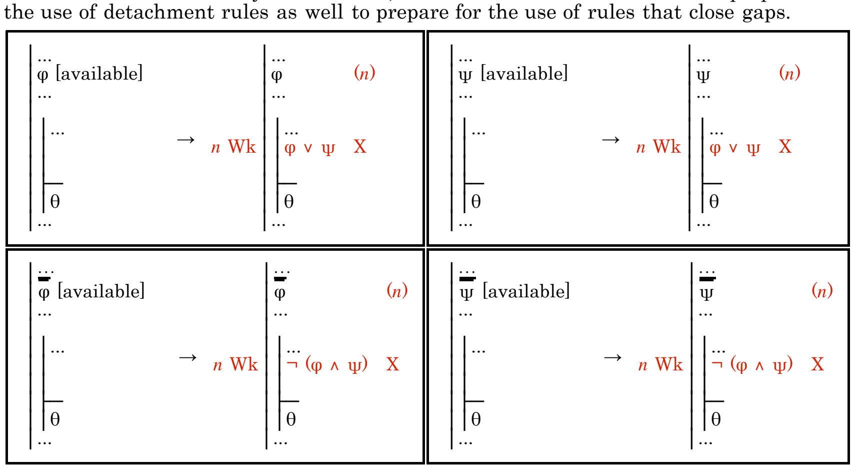
4.3.s. Summary

4.3.1. While a disjunction does not settle the truth values of its disjuncts, it says enough about them that adding the information that one is false will tell us that the other is true. This principle is known traditionally as *modus tollendo ponens*. Since each disjunct entails the disjunction, we know that, if one disjunct is false, then the disjunction and the other disjunct provide the same information. This idea is implemented in a further rule for exploiting disjunctions, also known as *Modus Tollendo Ponens* (MTP). The *not-both* form $\neg(\varphi \wedge \psi)$ is analogous to disjunction and analogous principles apply. Specifically, a principle *modus ponendo tollens* tells us that $\neg(\varphi \wedge \psi)$ together with the assertion of one of φ and ψ entails the denial of the other. And, since the denial of either φ or ψ entails $\neg(\varphi \wedge \psi)$, we can have a rule *Modus Ponendo Tollens* (MPT) for exploiting *not-both* forms. The rules MTP and MPT are examples of detachment rules. The resource exploited in each is its main resource and the additional resource that must be available is the auxiliary resource.



4.3.2. We will refer to as weakening the principle that disjunctions and *not-both* forms are entailed by assertions of components (in the case of disjunctions) or their denials (in the case of the *not-both* form). This principle provides the basis for two further attachment rules, both called Weakening (Wk), that license the addition of inactive resources. Since the second resource we must have in order to apply a detachment rule need only be available, attachment rules can be used to prepare for the use of detachment rules as well to prepare for the use of rules that close gaps.



Rules for developing gaps

	for resources	*for goals*
atomic sentence		IP
negation $\neg\varphi$	CR (if φ is not atomic and the goal is $\bot$)	RAA
conjunction $\varphi \wedge \psi$	Ext	Cnj
disjunction $\varphi \vee \psi$	PC	PE

Rules for closing gaps

when to close	*rule*
the goal is also a resource	QED
sentences φ and $\neg\varphi$ are resources & the goal is $\bot$	Nc
$\top$ is the goal	ENV
$\bot$ is a resource	EFQ

Basic system

Detachment rules (optional)

main resource	*auxiliary resource*	*rule*
$\varphi \vee \psi$	$\bar{\varphi}$ or $\bar{\psi}$	MTP
$\neg(\varphi \wedge \psi)$	φ or ψ	MPT

Added rules (optional)

Attachment rules

added resource	*rule*
$\varphi \wedge \psi$	Adj
$\varphi \vee \psi$	Wk
$\neg(\varphi \wedge \psi)$	Wk

Rule for lemmas

prerequisite	*rule*
the goal is $\bot$	LFR

4.3.x. Exercises

Redo the exercises of 4.2.x, looking for opportunities to use the new rules. (Each of the answers in 4.2.xa has as least one alternative using the new rules, and most of these new derivations are much shorter than the one given in the last section.)

1. Use derivations to establish each of the claims of entailment and equivalence shown below. (Remember that claims of equivalence require derivations in both directions.)

a. $A \wedge B \Rightarrow A \vee B$ **b.** $A \wedge B \Rightarrow B \vee C$ **c.** $A \vee B, \neg A \Rightarrow B$
d. $A \vee (A \wedge B) \Rightarrow A$ **e.** $A \vee B, \neg(A \wedge C), \neg(B \wedge C) \Rightarrow \neg C$ **f.** $A \wedge (B \vee C) \Rightarrow (A \wedge B) \vee C$
g. $A \vee B, C \Rightarrow (A \wedge C) \vee (B \wedge C)$ **h.** $A \vee B, \neg A \vee C \Rightarrow B \vee C$ **i.** $A \Leftrightarrow (A \wedge B) \vee (A \wedge \neg B)$

2. Use derivations to establish each of the claims of equivalence below.

a. $A \vee A \Leftrightarrow A$ **b.** $A \vee B \Leftrightarrow B \vee A$ **c.** $A \vee (B \vee C) \Leftrightarrow (A \vee B) \vee C$
d. $A \vee (B \wedge \neg B) \Leftrightarrow A$ **e.** $\neg(A \vee B) \Leftrightarrow \neg A \wedge \neg B$ **f.** $\neg(A \wedge B) \Leftrightarrow \neg A \vee \neg B$

3. Use derivations to check each of the claims below; if a derivation indicates that a claim fails, present a counterexample that divides an open gap.

a. $A \vee B, A \Rightarrow \neg B$ **b.** $A \vee (B \wedge C) \Leftrightarrow (A \vee B) \wedge C$ **c.** $\neg(A \vee B) \Leftrightarrow \neg A \vee \neg B$

Topics for test 2

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

- *Definitions of basic concepts.* Be able to state (in terms of possible worlds and truth values) the conditions under which sentences are mutually exclusive, jointly exhaustive, or contradictory and also the conditions under which the relation of relative exhuastiveness holds between sets.
- *Analysis.* Be able to analyze the logical form of a sentence as fully as possible using negation and disjunction in addition to conjunction and present the form in both symbolic and English notation (that is, with the logical and symbol and by expressing forms using `both–and`, etc.).
- *Derivations.* Be able to construct derivations to show that entailments hold and to show that they fail. I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. There will be at least one derivation where detachment and attachment rules may be used and where they will shorten the proof. But there will be other derivations where you must rely on others rules, either because detachment and attachment rules do not apply or because I tell you not to use them.