3.5.s. Summary

3.5.1. Any step in a derivation that is allowed by the basic rules (that is, for now, all rules except LFR and Adj) is safe and will take the derivation some way towards completion. We call the system of derivations limited to those rules the basic system. There will often be different orders in which the basic rules can be applied, and such differences may lead to longer or shorter derivations. The use of non-basic rules can sometimes shorten derivations still further, but they may not bring a derivation any closer to is final state.

The following table collects all rules we have now seen (and, as with the table of 2.4.s, the rule labels are links to the original statements of the rules):

Rules for developing gaps

	for resources	*for goals*
atomic sentence		IP
negation $\neg \phi$	CR (if ϕ is not atomic and the goal is $\bot$)	RAA
conjunction $\phi \wedge \psi$	Ext	Cnj

Rules for closing gaps

when to close	*rule*
the goal is also a resource	QED
sentences ϕ and $\neg \phi$ are resources & the goal is $\bot$	Nc
$\top$ is the goal	ENV
$\bot$ is a resource	EFQ

Basic system

Attachment rule

added resource	*rule*
$\phi \wedge \psi$	Adj

Added rules (optional)

Rule for lemmas

prerequisite	*rule*
the goal is $\bot$	LFR

3.5.x. Exercise questions

1. For each of the claims of entailment shown below, construct a derivation using only the basic rules and annotate it to show explicitly how it is the result of following the procedure given in 3.5.3. Provide one note for each pass through the procedure—i.e., one note for each stage followed by one for the final pass through the procedure that confirms that the derivation is done. Each note should indicate (i) the open gap chosen (or the fact that all gaps are closed), (ii) the proximate argument of this gap and either the rule (or rules) by which it may be closed or the rule (or rules) that may be applied to develop it, and (iii) whether the gap is closed, developed, or marked as a dead end (together with the rule used if there was a choice).
   - **a.** $\neg A \Rightarrow \neg (B \wedge A)$
   - **b.** $A \wedge B \Rightarrow B \wedge A$
   - **c.** $B \Rightarrow B \wedge A$
   - **d.** $\neg (A \wedge B), A \Rightarrow \neg B$
   - **e.** $\neg (A \wedge B), \neg (B \wedge C) \Rightarrow \neg B$

2. More than one derivation using the basic rules can be constructed for each of the claims of entailment below. In each case construct two and also recognize any further possibilities by noting each stage at which there was a choice between different ways of developing the derivation.
   - **a.** $A \wedge B \Rightarrow B \wedge A$
   - **b.** $\neg (A \wedge B), B \wedge C \Rightarrow \neg A$
   - **c.** $\neg (A \wedge B), \neg (B \wedge C) \Rightarrow \neg B$

Homework assigned Mon 9/26 and due Wed 9/28

Construct a derivation for the following claim of entailment and, for each stage, tell (i) which gap you chose to work on, (ii) the proximate argument of the gap and the (basic) rules that could have been applied, and (iii) whether you closed the gap, developed it, or marked it as a dead end (and the rule you applied if there was a choice): $\neg (A \wedge B), B \wedge C \Rightarrow C \wedge \neg A$