2.4.s. Summary

2.4.1.The introduction of a lemma is one way of dividing up the work of a proof. We can implement this idea in derivations by dividing a gap into two, with one child having the lemma as a goal and the other having it as a further assumption to use in reaching the goal of the parent gap. The rule Lemma (Lem) that does this is not safe in general nor is it always progressive, and we will use only special instances of it.

2.4.2. A lemma is always safe when it is entailed by the goal it is designed to help us reach. The principal use of this idea will come in arguments whose goal is $\perp$—that is, in *reductio* arguments. Since $\perp$ entails any sentence a rule Lemma for *Reductio* (LFR), which allows free use of lemmas in *reductio* arguments will be safe (though some restriction on its use is needed to insure it is progressive).

2.4.3. A lemma is also safe if it is entailed by things we already know. Rules applying this idea will be designed for particular sorts of entailment and, since such a lemma is known to follow from our resources, there is no need to divide the gap or even introduce a new scope line. Indeed, we will use this sort of lemma only in attachment rules that add the lemma as an available but inactive resource. The first example of this sort of rule is Adjunction (Adj) which adds a conjunction when both conjuncts are already available. Although attachment rules can help us to close gaps sooner, the rules themselves are not direct, and some care is needed in their use if they are to be progressive.

Summary of rules. The derivation rules we have so far are summarized in the table below. For the actual form taken by the rules, look at 2.2.3 in the case of Ext, Cnj, and QED, 2.2.5 in the case of ENV and EFQ, 2.4.2 in the case of LFR, and 2.4.3 in the case of Adj (the online version of this has links to the exact locations).

Rules for developing gaps			*Rules for closing gaps*	
	for resources	*for goals*	*when to close*	*rule*
conjunction $\phi \wedge \psi$	Ext	Cnj	the goal is also a resource	QED
			$\top$ is the goal	ENV
			$\perp$ is a resource	EFQ

Basic system

Attachment rule		*Added rules (optional)*
added resource	*rule*	
$\phi \wedge \psi$	Adj	

Rule for lemmas	
prerequisite	*rule*
the goal is $\perp$	LFR

2.4.x. Exercise questions

Use the basic system of derivations along with the attachment rule Adj to establish the following. These repeat entailments from earlier exercises and examples (specifically, **b** and **d** of exercise 2.2.x.2, exercises **2** and **4** of 2.3.x, and the example of 2.4.2). They will work best as exercises in the use of Adj if you avoid using Cnj.

1. $A \Rightarrow A \wedge A$
2. $A \wedge B, B \wedge C, C \wedge D \Rightarrow A \wedge D$
3. $A \wedge B \Rightarrow A \wedge (B \wedge A)$
4. $A, B \wedge C, D \Rightarrow (C \wedge (B \wedge A)) \wedge B$
5. $A \wedge B \Rightarrow (B \wedge A) \wedge (A \wedge (B \wedge A))$

Topics for test 1

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

- *Basic concepts of deductive logic*. You will be responsible for entailment (or validity) and implication, equivalence, tautologousness, absurdity, and inconsistency. You should be able to define each in terms of possible worlds and truth values, and you should be prepared to answer questions about them, justifying your answer by reference to the definitions. (You can find the definitions in 1.4 and also in Appendix A.1.)
- *Implicature*. Be able to define it and distinguish it from implication. Be able to give examples and explain them. Be ready to answer questions about it, justifying your answer by reference to its definition.
- *Analysis*. Be able to analyze the logical form of a sentence as fully as possible using conjunction and present the form in both symbolic and English notation (that is, with the logical-and symbol $\wedge$ and with the *both-and* way of expressing forms).
- *Derivations*. Be able to construct derivations to show that entailments hold and to show that they fail. I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. There may be some derivations where the rule Adj introduced in 2.4 would be convenient to use; but it is never necessary. You should be ready to use EFQ and ENV as well as Ext, Cnj, and QED; but derivations involving the latter three are much more likely.