

1.2.s. Summary

1.2.1. The relation of entailment concerns the possibilities of truth and falsity for premises and conclusions; that is, it concerns the **truth values** of these sentences in various **possible worlds**. The possibilities in question are **logical possibilities**, which may be understood as those situations whose description is permitted by the semantic rules of the language.

1.2.2. Information about the truth values of a sentence in all possible worlds is information about its **truth conditions**, and these truth conditions determine the **proposition** it expresses. Sentences that have the same truth conditions, that express the same proposition, are **logically equivalent** (an idea for which we use the sign \Leftrightarrow). From the point of view of deductive logic, equivalent sentences have the same properties and stand in the same relations to other sentences. Entailment is one relation among sentences that depends only on the propositions they express. A conclusion ϕ is entailed by a set Γ of premises when ϕ rules out only worlds that are ruled out by at least one member of Γ . This is a way of saying that the content of ϕ does not exceed that of the members of Γ taken together, so entailment is a comparison of sentences in terms of their informational content. At one extreme are **tautologies**, which rule out no possibilities and thus have no content. All tautologies are equivalent and we will distinguish one, **Tautology**, for which we use the notation \top . At the other extreme are sentences that rule out all possibilities. Such sentences are **absurd** and all are equivalent to the single representative **Absurdity**, for which we use the notation \perp .

1.2.3. Although certain groups of sentences can be ordered linearly between \perp and \top as a series of claims with steadily increasing content, the full range of propositions expressed by sentences are better thought of as inhabiting a much more complex **logical space**. This can be thought of, on the one hand, as a space of possibilities, with an individual proposition constituting a division of the space into two regions, the possibilities it rules out and the possibilities it leaves open. Another sort of space has as its points not possible worlds but propositions, with different possible worlds representing different dimensions with respect to which the location of propositions can differ. Logical space in this sense has a bottom in the proposition expressed by \perp and a top provided by \top . So long as there are alternative possibilities (that is, more than just one possible world), there will be more propositions with intermediate content than there are degrees of content intermediate between \perp and \top .

1.2.4. This picture of deductive reasoning fits into a simplified picture of the function of language. Our beliefs, the information we think we have, amount to a proposition that rules out a certain range of possibilities for the history of the universe. In general, we would like to narrow down the range of possibilities left open even further. When language is used cooperatively (something that must be the standard case), we share the ability to rule out possibilities by

asserting sentences that rule out some of the possibilities our beliefs lead us to exclude. The sentences we can sincerely assert are the ones that express propositions that are entailed by the proposition expressing the sum total of our beliefs.

1.2.x. Exercise questions

- Suppose you know that a certain argument is valid but do not know whether its premises and conclusion are true or false. If you are given one of the further items of information **a-c** about the premises of the argument, what if anything can you say about the truth value of its conclusion?
 - The premises are all true.
 - The premises are all false.
 - Some premises are true and some are false.
- Suppose that $\phi, \psi / \chi$ is an argument that you know to be valid. If you find that the conclusion χ is false, what if anything can you say about the truth values of the premises ϕ and ψ ?
- For each of the following items of information, tell what you can conclude from it about the equivalence of sentences ϕ and ψ .
 - ϕ and ψ are both true
 - ϕ and ψ are both false
 - ϕ is true and ψ is false
 - There is a sentence χ such that χ and ϕ together entail ψ , and χ and ψ together entail ϕ (i.e., $\chi, \phi \Rightarrow \psi$ and $\chi, \psi \Rightarrow \phi$)
- For each of the following pieces of information, tell what if anything you can conclude about the possibilities left open and the possibilities ruled out by the sentence ϕ :
 - ϕ is equivalent to a tautology ψ
 - ϕ entails \top
 - a tautology ψ entails ϕ
 - ϕ is equivalent to \perp
 - ϕ entails an absurdity ψ
 - \perp entails ϕ

Homework due Wed 8/31

Give two definitions of the claim of entailment $\phi, \psi, \chi \Rightarrow \theta$ in terms of truth values and possible worlds—one in negative form and the other in positive form.