8.4. Definite descriptions

8.4.0. Overview

Up to this point, we have analyzed definite descriptions only by identifying component individual terms; now we will consider two ways of analyzing them to identify the descriptions from which they are formed.

8.4.1. Definite descriptions as quantifier phrases
On one approach, the definite description *the* *X* is a quantifier phrase that differs from *a* *X* by adding the claim *there is at most one* *X*.

8.4.2. Definite descriptions as individual terms
On another analysis, which yields a different account of their logical properties, definite descriptions are formed by an operation that applies to predicates to yield individual terms.

8.4.3. Examples: restrictive *vs.* non-restrictive relative clauses
The analysis of definite descriptions makes it possible to represent the distinction between restrictive and non-restrictive relative clauses in the case of definite descriptions.

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8.4.1. Definite descriptions as quantifier phrases

We have been treating definite descriptions as individual terms and analyzing them only by extracting component terms. In the early years of this century the British logician and philosopher Bertrand Russell (1872-1970) proposed a way of analyzing definite descriptions that, in effect, treats them as quantifier phrases. For example, he would treat the sentence *The house Jack built still stands* as making a claim that could be stated more explicitly as:

Something such that it and only it is house Jack built is such that (it still stands)

If we make this restatement the starting point of a symbolic analysis, we will get the following:

The house Jack built still stands
Something such that it and only it is house Jack built is such that (it still stands)
(∃x: x *and only* x *is a house Jack built*) x *still stands*
(∃x: x *is a house Jack built* ∧ *only* x *is a house Jack built*) Sx
(∃x: (x *is a house* ∧ *Jack built* x) ∧ *only a thing identical to* x *is such that (it is a house Jack built)*) Sx
(∃x: (Hx ∧ Bjx) ∧ (∀y: ¬ y = x) ¬ y *is a house Jack built*) Sx
(∃x: (Hx ∧ Bjx) ∧ (∀y: ¬ y = x) ¬ (y *is a house* ∧ *Jack built* y)) Sx

(∃x: (Hx ∧ Bjx) ∧ (∀y: ¬ y = x) ¬ (Hy ∧ Bjy)) Sx
∃x ((Hx ∧ Bjx) ∧ ∀y (¬ y = x → ¬ (Hy ∧ Bjy)) ∧ Sx)

[B: λxy (x *built* y); H: λx (x *is house*); S: λx (x *still stands*); j: *Jack*]

Notice that the sentence *A house Jack built still stands* could be restated as *Something such that it is house Jack built is such that (it still stands)*, so the difference between the indefinite and definite article on Russell's analysis lies in the extra phrase *and only if*. In the analysis above, that phrase yields an added conjunct in the restricting formula that appears in English as *only* x *is a house Jack built* and in symbols as (∀y: ¬ y = x) ¬ (Hy ∧ Bjy). This reflects the requirement of uniqueness noted in 6.2.1 as a condition for the reference of definite descriptions, the analysis above entails *Jack built at most one house*.

Notice that Russell does not treat *The house Jack built still stands* as *Exactly one house Jack built still stands*. The latter sentence makes a claim of unique, too, but a weaker one. It entails only *Jack built at most one house that still stands* and not *Jack built at most one house*. Russell's analysis also entails *Any house Jack built still stands* and this means that, with a little artificiality, the difference between it and the weaker claim of uniqueness can be expressed as the difference between a non-restrictive and a restrictive relative clause--i.e., between the stronger *The houses Jack built, which still stand, number one* and the weaker *The houses Jack built that still stand number one*. Notice that the first of these cannot be treated as a simple claim that there is exactly one example of a certain sort.

In general, Russell recommended that we analyze a sentence of the form *The* C *is such that (... it ...)* as equivalent to

(∃x: x *is a* C ∧ (∀y: ¬ y = x) ¬ y *is a* C) ... x ...

--i.e., as we might analyze *Something such that it and only it is a* C *is such that (... it...)*. It is sometimes convenient to use instead the shorter form

(∃x: x *is a* C ∧ (∀y: y *is a* C) x = y) ... x ...

which amounts to *Some* C *that is all the* C*s there are is such that (... it...)*. As was noted in 8.3.3 for a similar restatement of sentences using *exactly 1*, this is equivalent to the first form by principle of contraposition and the symmetry of identity.

This pattern of analysis has often been followed since, but it is not uncontroversial, since it opens up the possibility of scope ambiguities that many do not find in sentences involving definite descriptions. In particular, when analyzing a negative sentence containing a definite description, we can regard the negation either as the main logical operator or as a part of the quantified predicate left when we remove the definite description. To choose one of Russell's own examples, we could regard *The present king of France is not bald* as making either of the claims below.

¬ *the present king of France is bald*
The present king of France is such that he is not bald

Russell's analysis of the positive claim *The present king of France is bald* implies that there is at present a king of France. So he holds that the first of the sentences above is true because it is the negation of a false statement. But, by the same token, the second sentence claims in part that there is presently a king of France, so it is false on his view. Thus *The present king of France is not bald* is, on Russell's analysis, open to two interpretations, one on which it is true and another on which it is false, and many philosophers have found no such ambiguity in the sentence. Indeed, many would claim that the sentence is neither true nor false since the definite description *the present king of France* does not refer to anything.

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8.4.2. Definite descriptions as individual terms

Prior to 8.4.1, we had treated definite descriptions as individual terms, understanding definite descriptions to have at least the nil value as a reference value. (A somewhat similar approach was favored by Frege. He suggested that an actual object--for example, the number 0--be stipulated as the reference of definite descriptions that did not otherwise have one.) This sort of approach to definite descriptions has not been explicit in our symbolic notation because we have so far left such expressions unanalyzed. To analyze definite descriptions while still treating them as individual terms, we can introduce a ***description operator***. This is an operation that applies to a predicate abstract to form an individual term. Our notation will be a sans-serif captial I and we will abbreviate I[λx ρx] as Ix ρx. This notation might be read in English as *the thing* x *such that* ρx. The reference value of Ix ρx is stipulated to be the one value in the extension of ρ if contains just one value and to be the nil value otherwise. We do not distinguish the nil value from others in a referential range in any other way, so our stipulation of it as the default value of Ix ρx is limited in its significance. This stipulation does entail that definite descriptions that fail to uniquely describe an object all have the same reference value.

If we use the description operator to analyze *The house Jack built still stands*, we get

The house Jack built still stands
S *the house Jack built*
S(Ix (x *is a house Jack built*))
S(Ix (x *is a house* ∧ *Jack built* x))

S(Ix (Hx ∧ Bjx))

[B: λxy (x *built* y); H: λx (x *is house*); S: λx (x *still stands*); j: *Jack*]

The parentheses surrounding the whole definite description in this analysis are not needed to avoid ambiguity in our notation, but they make it easier to read when we use the same conventions for spacing that we followed earlier.

This analysis does more than use different notation from Russell's analysis; it offers a different interpretation of the sentence. While the simpler notation may be pleasing, the interpretation may not be, so we should consider it more closely. To compare the two interpretations, it will help to give Russell's in a different but equivalent form. Since on Russell's analysis *The* C *is such that (... it ...)* entails both *Some* C *is such that (... it ...)* and that at most one thing is a C, it can be restated somewhat redundantly as the conjunction

There is exactly one C ∧ *some* C *is such that (... it ...)*

That is, Russell interprets *The house Jack built still stands* as *There is exactly one house that Jack built and some house that Jack built still stands*.

On the other hand, if we analyze *The* C *is such that* (... *it* ...) using the description operator, we interpret it as saying that the predicate λx (... x ...) is true of the reference value of *the* C. Now, what that reference values is depends on whether *There is exactly one* C is true. If there is exactly one C, the value of *the* C is the one and only C. Otherwise, the value of *the* C is the nil value.

To make it easier to express this in English, let's fix an individual term whose reference is bound to be nil and read it in English as *the nil*. Since the extension of $\lambda x \perp$ is bound to be empty, the definite description ℩x $\perp$ could play this role, but it will be convenient to have a special symbol, for which we will use $*$ (known as the ***asterisk operator***).

Putting all this together , we can express the content of the analysis using the description operator as follows:

(*there is exactly one* C $\wedge$ *some* C *is such that* (... *it* ...))
$\vee$ ($\neg$ *there is exactly one* C $\wedge$ (... *the nil* ...))

Comparison with the expression of Russell's analysis given above will show that this interpretation is weaker, having been hedged by an added disjunct. It could be expressed equivalently as follows:

If there is exactly one C, *then some* C *is such that* (... *it* ...);
otherwise (... *the nil* ...)

where the English *if* ϕ *then* ψ *; otherwise* χ expresses the form $(\phi \rightarrow \psi) \wedge (\neg \phi \rightarrow \chi)$, which might be called a ***branching conditional***. This is equivalent to the form $(\phi \wedge \psi) \vee (\neg \phi \wedge \chi)$ that was used above because each form has the same truth value as ψ when ϕ is true and the same value as χ when ϕ is false. The second of formulations displayed above makes the comparison with Russell's analysis a little less direct but it is the more natural way of thinking about the significance of this approach to definite descritpions in its own right.

On this approach, then, we interpret *The house Jack built still stands* as either of the following equivalent claims:

Either there is exactly one house that Jack built and some house that Jack built still stands;
or there is not exactly one and the nil still stands
If there is exactly one house that Jack built then some house that Jack built still stands;
otherwise the nil still stands

This interpretation has both fortunate and unfortunate consequences.

First the bad news. Because the analysis using the description operator hedges the claim it makes with the possibility that there is not exactly one house that Jack built, it can be true if he built no house or more than one. So we must ask whether we would count the original sentence as true in this sort of case. In answering this question, it is important to remember that the analysis will be true in such a case only if the predicate λx (x *still stands*) is true of the nil reference value. The truth value yielded by properties when they are applied to the nil value is something that we have left open. (More precisely, this is true in the case of unanalyzed predicates; $\lambda x\, x = x$, for example, is bound to be true of the nil value because it is true of all reference values.) So when we analyze definite descriptions using the description operator, we do not specify the truth value of *The house Jack built still stands* in cases where *the house Jack built* does not refer. But on Russell's account the value is definitely **F** in these cases. If the discussion of the issue throughout the course of this century has shown anything, it has shown that there is no consensus on this matter among the community of English speakers.

That's the bad news. The good news is that the analysis using the description operator removes any room for ambiguity concerning the relative scope of definite descriptions and negation. That much is clear just from the notation. The definite description operator forms terms and to deny that a predicate applies to a term is the same thing as to apply a negative predicate. That is, $\neg\, \theta\, \tau \Leftrightarrow [\lambda x \neg\, \theta x]\, \tau$. (Indeed, we really have more than an equivalence here since we regard these symbolic forms as notation for the same sentence.)

We can see this lack of ambiguity also by exploring the interpretation given by the second analysis. On Russell's analysis, *The present king of France is not bald* exhibits a scope ambiguity that can be exhibited in the following restatements and partial analyses of a pair of sentences:

The present king of France is such that (he is bald)
There is at present one and only one king of France
$\wedge$ *some present king of France is such that (he is bald)*

O $\wedge$ ($\exists$x: Kx) Bx
The present king of France is such that (he is not bald)
There is at present one and only one king of France
$\wedge$ *some present king of France is such that (he is not bald)*

O $\wedge$ ($\exists$x: Kx) $\neg$ Bx
[B: λx (x *is bald*); K: λx (x *is at present king of France*);
O: *there is at present one and only one king of France*]

If O is true, at least one of these is true because there is some king of France at present who must be either bald or not and at most one is true because there is no more than one present king of France so being bald and not being bald cannot both be exemplified by present kings of France. But if O is not true both are false, so they are not contradictory. Now, on Russell's analysis, *The present king of France is not bald* might be analyzed as equivalent to either O $\wedge$ ($\exists$x: Kx) $\neg$ Bx or $\neg$ (O $\wedge$ ($\exists$x: Kx) Bx) and, because the two sentences above are not contradictory, these two interpretations are not equivalent.

On the other hand if we consider the same two sentences but restate them in the way corresponding to the semantics of the definite description operator we get this:

The present king of France is such that (he is bald)
(O $\wedge$ *some present king of France is such that (he is bald)*) $\vee$ ($\neg$ O $\wedge$ *the nil is bald*)

(O $\wedge$ ($\exists$x: Kx) Bx) $\vee$ ($\neg$ O $\wedge$ B$*$)
The present king of France is such that (he is not bald)
(O $\wedge$ *some present king of France is such that (he is not bald)*) $\vee$ ($\neg$ O $\wedge$ *the nil is not bald*)

(O $\wedge$ ($\exists$x: Kx) $\neg$ Bx) $\vee$ ($\neg$ O $\wedge$ $\neg$ B$*$)

Now, we have already seen that, if O is true, the left disjunct of exactly one of these is true and, since the right disjuncts are both false when O is true, exactly one of the disjunctions will be true in such a case. And, when O is false, the left disjuncts are both false and exactly one of the right disjuncts is true. So again exactly one the disjunctions is true, and these sentences are contradictory. Thus it doesn't matter whether *The present king of France is not bald* is understood as equivalent to the first or as equivalent to the negation of the second.

In an analysis using the description operator, both sentences are given weaker interpretations than Russell would give them and these interpretations are weaker in different ways. In particular, in a case where O is false, one of the hedges is true and the other is not. Which is which depends on whether λx (x *is bald*) is true or false of the nil value--but we do not care which hedge is true and which false, only that they have different values when the left disjuncts are both false.

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8.4.3. Examples: restrictive *vs.* non-restrictive relative clauses

The distinction between restrictive and non-restrictive relative clauses is a natural object of study for the analysis of definite descriptions. Although the significance of the distinction is not as great as it is with generalizations, it does have some significance, unlike the case of claims of exemplification. And since restrictive relative clauses are part of definite descriptions but not themselves individual terms, it is only the sort of analysis of definite descriptions that we are now considering that can exhibit their role.

We will consider a single pair of sentences and analyze each of them using the two approaches to definite descriptions. Since these analyses are not equivalent, we can expect different results but, since the difference between the analyses involves a failure of normal reference, we cannot expect great differences when work normally, when reference succeeds and true claims are made.

The two sentences we will consider are these:

The part that Tom requested was defective.
The part, which Tom requested, was defective.

The difference between having a restrictive relative clause in the first and a non-restrictive relative clause in the second is, intuitively, whether the relative clause contributes to the specification of what is referred to or instead to what is said about it. That difference would be sharper still where the second sentence to be expanded to *The part, which, by the way, Tom requested, was defective*.

We will begin with an analysis of these two sentences using the description operator. This begins as analysis in chapter 6 would have but continues further. In the case of the first sentence, we have

The part that Tom requested was defective
The part that Tom requested *was defective*
D *the part that Tom requested*
D(lx x *is a part that Tom requested*)
D(lx (x *is a part* ∧ *Tom requested* x))

D(lx (Px ∧ Rtx))

[D: λx (x *was defective*); P: λx (x *is a part*); R: λxy (x *requested* y); t: *Tom*]

In chapter 6, we would have ended up with something like D(pt) where p abbreviated a functor that produced the term *the part that Tom requested* when applied to the term *Tom*. Since the two expressions

[λy (lx (Px ∧ Ryx))]t   lx (Px ∧ Rtx)

are really two forms of notation for the same term, we can say that the analysis above extends the analysis of chapter 6 by analyzing the functor p as λy (lx (Px ∧ Ryx)).

We also begin analyzing the sentence with non-restrictive relative clause as before, treating it as a conjunction.

The part, which Tom requested, was defective
Tom requested the part ∧ *the part was defective*
R *Tom the part* ∧ D *the part*
$Rt(\text{l}x\ x\ \textit{is a part}) \land D(\text{l}x\ x\ \textit{is a part})$

$$Rt(\text{l}x\ Px) \land D(\text{l}x\ Px)$$

$[D\!: \lambda x\,(x\ \textit{was defective});\ P\!: \lambda x\,(x\ \textit{is a part});\ R\!: \lambda xy\,(x\ \textit{requested}\ y);\ t\!: \textit{Tom}]$

To make it easier to compare the two analyses, let us reorder the conjuncts in the second to get

$$D(\text{l}x\ Px) \land Rt(\text{l}x\ Px)$$

and then restate this using an abstract so that the definite description occurs only once

$$[\lambda x\,(Dx \land Rtx)](\text{l}x\ Px)$$

The difference between the sentences restrictive and non-restrictive clauses, when seen in this way--i.e., as

$$D(\text{l}x\ (Px \land Rtx)) \qquad [\lambda x\,(Dx \land Rtx)](\text{l}x\ Px)$$

--lies in the use of the predicate $\lambda x\ Rtx$ or $\lambda x\,(\textit{Tom requested}\ x)$ to provide a further conjunct either in the description to which the definite article is applied or in what is predicated of a definite description. This is the symbolic analogue of the idea that restrictive relative clause contributes to determining the reference of an individual term while a non-restrictive clause adds to what is said about the term's referent.

We can expect to find something similar when apply Russell's analysis. In the case of the first sentence, we get

The part that Tom requested was defective
The part that Tom requested is such that (it was defective)
$(\exists x\!: x\ \textit{is a part that Tom requested} \land (\forall y\!: \neg\, y = x)\ \neg\, y\ \textit{is a part the Tom requested})\ x\ \textit{was defective}$
$(\exists x\!: (x\ \textit{is a part} \land \textit{Tom requested}\ x) \land (\forall y\!: \neg\, y = x)\ \neg\,(y\ \textit{is a part} \land \textit{Tom requested}\ y))\ x\ \text{was defective}$

$(\exists x\!: (Px \land Rtx) \land (\forall y\!: \neg\, y = x)\ \neg\,(Py \land Rty))\ Dx$
or: $(\exists x\!: (Px \land Rtx) \land (\forall y\!: Py \land Rty)\ x = y)\ Dx$

$[D\!: \lambda x\,(x\ \textit{was defective});\ P\!: \lambda x\,(x\ \textit{is a part});\ R\!: \lambda xy\,(x\ \textit{requested}\ y);\ t\!: \textit{Tom}]$

On Russell's analysis, the definite article *the* can be seen to mark an operation that applies to a predicate ρ to form the quantifier $(\exists x\!: \rho x \land (\forall y\!: \neg\, y = x)\ \neg\, \rho y)$ or $(\exists x\!: \rho x \land (\forall y\!: \rho y)\ x = y)$. In the sentence with the restrictive relative clause the predicate ρ is $\lambda x\,(Px \land Rtx)$ and it employs the predicate $\lambda x\ Rtx$ corresponding to the relative clause.

We can expect Russell's analysis of the sentence with a non-restrictive relative clause to find a conjunction. But we must choose whether this conjunction has wider or narrower scope than the quantifier phrase associated with the definite description. This is the sort of thing that leads to non-equivalent analyses in the case of negation, but here the results of the two analyses shown below are equivalent.

The part, which Tom requested, was defective
Tom requested the part ∧ *the part was defective*
the part is such that (Tom requested it) ∧ *the part is such that (it was defective)*
$(\exists x\!: x\ \textit{is a part} \land (\forall y\!: \neg\, y = x)\ \neg\, y\ \textit{is a part})\ \textit{Tom requested}\ x \land (\exists x\!: x\ \textit{is a part} \land (\forall y\!: \neg\, y = x)\ \neg\, y\ \textit{is a part})\ x\ \textit{was defective}$

$(\exists x\!: Px \land (\forall y\!: \neg\, y = x)\ \neg\, Py)\ Rtx \land (\exists x\!: Px \land (\forall y\!: \neg\, y = x)\ \neg\, Py)\ Dx$
or: $(\exists x\!: Px \land (\forall y\!: Py)\ x = y)\ Rtx \land (\exists x\!: Px \land (\forall y\!: Py)\ x = y)\ Dx$

$[D\!: \lambda x\,(x\ \textit{was defective});\ P\!: \lambda x\,(x\ \textit{is a part});\ R\!: \lambda xy\,(x\ \textit{requested}\ y);\ t\!: \textit{Tom}]$
The part, which Tom requested, was defective
The part is such that (it, which Tom requested, was defective)
$(\exists x\!: x\ \textit{is a part} \land (\forall y\!: \neg\, y = x)\ \neg\, y\ \textit{is a part})\ x,\ \textit{which Tom requested, was defective}$
$(\exists x\!: Px \land (\forall y\!: \neg\, y = x)\ \neg\, Py)\ (\textit{Tom requested}\ x \land x\ \textit{was defective})$

$(\exists x\!: Px \land (\forall y\!: \neg\, y = x)\ \neg\, Py)\ (Rtx \land Dx)$
or: $(\exists x\!: Px \land (\forall y\!: Py)\ x = y)\ (Rtx \land Dx)$

In general we cannot expect an existential applied to a conjunction to be equivalent to the result of apply the existential to each conjunct; the sentence *Someone lives in London and works in Singapore* says a good deal more than the conjunction *Some lives in London and someone works in Singapore*. The reason for the difference in such cases is that different examples may make each conjunct of the conjunction true and there may be no one example that would serve for both, which is what would be required for the first example to be true. But this problem will not arise in the case of the sort of existentials used by Russell to represent definite descriptions because the restricting formula $\rho x \land (\forall y\!: \neg\, y = x)\ \neg\, \rho y$, or the alternative form $\rho x \land (\forall y\!: \rho y)\ x = y$, says that x is the one and only thing in the extension of ρ. This means that if we claim the existence of an example x that satisfies this restricting formula, part of what we have claimed is that this is the only example possible. So if, we claim, for each of two properties, that it is exemplified by something x that satisfies this restricting formula, part of what we are saying is that the example is the same for each property; therefore, our claim will entail the existence of a single example with both properties.

Here is a table showing the simplest analyses of the four sorts:

	restrictive clause	*non-restrictive clause*
description operator	$D(\text{l}x\ (Px \land Rtx))$	$Rt(\text{l}x\ Px) \land D(\text{l}x\ Px)$
Russell's analysis	$(\exists x\!: (Px \land Rtx) \land (\forall y\!: Py \land Rty)\ x = y)\ Dx$	$(\exists x\!: Px \land (\forall y\!: Py)\ x = y)\ (Rtx \land Dx)$

If we convert Russell's analyses to unrestricted existential quantifiers and reorder conjuncts, we get the following:

restrictive clause	$\exists x\, (Px \land Rtx \land Dx \land (\forall y\colon Py \land Rty)\, x = y)$
non-restrictive clause	$\exists x\, (Px \land Rtx \land Dx \land (\forall y\colon Py)\, x = y)$

This means that the sentence stated using the restrictive relative clause is entailed by the sentence using the non-restrictive clause, since the former ascribes the example it claims to exist a more restricted general property, saying about this example only that it accounts for all the parts Tom requested rather all the parts whatsoever. By the same token it is clear that no entailment holds in the other direction: if there was more than one part but only one that Tom requested and that part was defective, the first sentence above is true but the second is false since nothing will be identical to every part.

These differences are easiest to describe for Russell's analysis because when the definite description has the whole sentence in its scope (as is true here), a failure of the description to be uniquely satisfied leads makes the sentence false. In analyses using the description operator, the way things fail is important because, if a description is not uniquely satisfied, the definite description has the nil reference value and whether what is said is true or false will depend on what predicates are true or false of this value. If the description $\iota x\, Px$ has a non-nil reference and the sentence $Rt(\iota x\, Px) \land D(\iota x\, Px)$ is true, then the description $\iota x\, (Px \land Rtx)$ will also have a non-nil reference and the sentence $D(\iota x\, (Px \land Rtx))$ will be true. If both descriptions have nil references the two sentences have the same truth values as $Rt* \land D*$ and $D*$, respectively, so the sentence with a restrictive relative clause will again be true if the sentence with a non-restrictive clause is. But it is possible for $\iota x\, (Px \land Rtx)$ to have a non-nil reference value when $\iota x\, Px$ has a nil value because there may be more than one part but only one that Tom requested. If D is false of the reference value of $\iota x\, (Px \land Rtx)$--i.e., the part that Tom requested was not defective--then the sentence using the restrictive relative clause is false, but the sentence using the non-restrictive clause will have the same truth value as $Rt* \land D*$ and this may be true. So, on the analyses using the description operator, the sentence using the non-restrictive clause does not entail the one using the non-restrictive clause.

It's hard to say which analysis gives the correct account of the logical relations of these two sentences because it is hard to make judgements of truth-values of sentences containing definite descriptions whose component descriptions are not uniquely satisfied. But it can be said that, although the argument in the case of the description operator may seem too ready to exploit the abitrariness of the truth values of predicates when they are applied to the nil, the possibility of predicates being true in such a case is closely tied to the unambiguity, on this analysis, of sentences like *The king of France is not bald*. Were we to say that every predicate is false when applied to the nil, we would be forced to say, in cases where there is more than one part, that *The part is not defective* is true when analyzed as the negation of $D(\iota x\, Px)$ but false when analyzed as $[\lambda x\, \neg Dx](\iota x\, Px)$--i.e., as the application of a negative predicate. If we were to say only that *unanalyzed* predicates are always false of the nil, we could count $[\lambda x\, \neg Dx](\iota x\, Px)$ as true but the failure of entailment between sentences with with the two sorts of clauses would reappear with sentences like *The part that Tom didn't request is not defective* and *The part, which Tom didn't request, is not defective*. When there was more than one part but only one that Tom didn't request and that part was not defective, the first of these sentences is false but the second is true because it has the same truth value as $\neg Rt* \land \neg D*$.

It should be added that the differences between restrictive and non-restrictve clauses are not only matters of truth values. Even if we suppose that a *The part that Tom requested was defective* is entailed by *The part, which Tom requested, was defective*, the former will not be appropriate in all contexts where the latter is. To be used appropriately, a definite description must do enough to identify a unique object given the context and it should usually do this using information that is already

available. So in a context where a particular part was already singled out as the most salient--so that *the part* was appropriate--but nothing has been said about Tom requesting a part, the definite description *the part that Tom requested* can seem odd on two counts. First, it suggests that the context was not enough to single out one part and that can lead the audience to wonder what range of parts the speaker has in mind. Second, it may prompt the response, "I didn't know Tom requested a part." That is, the use of description containing the relative clause presupposes information that was not already on the table rather than providing this information, as the sentence with the non-restrictive clause can be understood to do.

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8.4.s. Summary

A famous analysis of definite descriptions was first proposed early in the 20th century by Bertrand Russell. According to Russell's analysis, a sentence *The* C *is such that (... it ...)* amounts to *Something such that it and only it is a* C *is such that (... it ...)*. This analysis is equivalent to the conjunction of *Some* C *is such that (... it ...)* and *There is at most one* C, so the effect of using a definite rather than an indefinite article is to imply the latter conjunct. Russell's analysis treats a definite description as a kind of quantifier phrase and leads to scope ambiguities in negative sentences involving definite descriptions.

An alternative approach avoids this suggestion of ambiguity by treating definite descriptions as individual terms and analyzing them by the use of a description operator, which applies to predicate abstracts to form terms. We use a sans-serif capital I as notation for the description operator, abbreviating $\mathsf{I}[\lambda x\ \rho x]$ by $\mathsf{I}x\ \rho x$. A term formed in this way has the sole member of the predicate's extension as its reference value if that extension has a unique member and otherwise its reference value is the nil value. We fix a logically constant term, the nil, which always has the nil value and use the notation $*$ (asterisk operator) for it. The content of *...* *the* C *...* on this analysis can be expressed using a branching conditional as *if there is exactly one* C*, then some* C *is such that (... it ...); otherwise, ... the nil ....*

Each of the two approaches to analyzing definite descriptions can be used to exhibit the difference between a restrictive and a non-restrictive relative clause when these modify a common governed by the article *the*. Although both analyses point to differences between such sentences, their accounts of the relations between them differ.

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8.4.x. Exercise questions

1. Analyze the following in as much detail as possible; analyze definite descriptions in two ways, using Russell's approach and using the description operator.

- **a.** *Sam guessed the winning number.*
- **b.** *The winner who spoke to Tom was well-known.*
- **c.** *The winner, who spoke to Tom, was well-known.*
- **d.** *Every number greater than one is greater than its (own) positive square root.*

2. Synthesize idiomatic English sentences that express the propositions associated with the logical forms below using the intensional interpretations that follow them. You may use definite descriptions to express the sort of logical forms Russell's analysis produces.

- **a.** $(\exists x\colon Oxs \land (\forall y\colon \neg\, y = x)\ \neg\, Oys)\ Cx$
  [C: λx (x *called*); O: λxy (x *owns* y); s: *Spot*]
- **b.** $Fj(\mathsf{I}x\ (Hx \land Ex(\mathsf{I}y\ Pyj)))$
  [E: λxy (x *enlarged* y); F: λxy (x *found* y); H: λx (x *is a photographer*); P: λxy (x *is a picture of* y); j: *John*]

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8.4.xa. Exercise answers

1. a. *using Russell's analysis:*

Sam guessed the winning number
the winning number is such that (Sam guessed it)
(∃x: x *is a winning number* ∧ *only* x *is a winning number*) *Sam guessed* x
(∃x: Wx ∧ (∀y: ¬ y = x) ¬ y *is a winning number*) Gsx

$$(\exists x\colon Wx \land (\forall y\colon \neg\, y = x)\, \neg Wy)\, Gsx$$
$$\exists x\, (Wx \land \forall y\, (\neg\, y = x \to \neg Wy) \land Gsx)$$
or:
$$(\exists x\colon Wx \land (\forall y\colon Wy)\, x = y)\, Gsx$$
$$\exists x\, (Wx \land \forall y\, (Wy \to x = y) \land Gsx)$$

[G: λxy (x *guessed* y); W: λx (x *is a winning number*); s: *Sam*]
[*Note:* λx (x *is a winning number*) might be open to further analysis as λx (x *is a number* ∧ x *won*)]

with the description operator:

Sam guessed the winning number
G *Sam the winning number*
Gs(lx x *is a winning number*)

Gs(lx Wx)

b. *using Russell's analysis:*

The winner who spoke to Tom was well-known
The winner who spoke to Tom is such that (he or she was well-known)
(∃x: x *is a winner who spoke to Tom* ∧ *only* x *is a winner who spoke to Tom*) x *was well-known*
(∃x: (x *is a winner* ∧ x *spoke to Tom*) ∧ (∀y: ¬ y = x) ¬ (y *is a winner* ∧ y *spoke to Tom*)) Kx

$$(\exists x\colon (Wx \land Sxt) \land (\forall y\colon \neg\, y = x)\, \neg (Wy \land Syt))\, Kx$$
$$\exists x\, ((Wx \land Sxt) \land \forall y\, (\neg\, y = x \to \neg (Wy \land Syt)) \land Kx)$$
or:
$$(\exists x\colon (Wx \land Sxt) \land (\forall y\colon Wy \land Syt)\, x = y)\, Kx$$
$$\exists x\, ((Wx \land Sxt) \land \forall y\, ((Wy \land Syt) \to x = y) \land Kx)$$

K: λx (x *was well-known*); S: λxy (x spoke to y); W: λx (x *is a winner*); t: *Tom*]

with the description operator:

The winner who spoke to Tom was well-known
The winner who spoke to Tom was well-known
K *the winner who spoke to Tom*
K(lx (x *is a winner who spoke to Tom*))
K(lx (x *is a winner* ∧ x *spoke to Tom*))

K(lx (Wx ∧ Sxt))

c. *using Russell's analysis:*

The winner, who spoke to Tom, was well-known.
The winner is such that (he or she, who spoke to Tom, was well-known).
(∃x: x *is a winner* ∧ *only* x *is a winner*) x, *who spoke to Tom, was well-known*)
(∃x: x *is a winner* ∧ (∀y: ¬ y = x) ¬ y *is a winner*) (x *spoke to Tom* ∧ x *was well-known*)

$$(\exists x\colon Wx \land (\forall y\colon \neg\, y = x)\, \neg Wy)\, (Sxt \land Kx)$$
$$\exists x\, (Wx \land \forall y\, (\neg\, y = x \to \neg Wy) \land (Sxt \land Kx))$$
or:
$$(\exists x\colon Wx \land (\forall y\colon Wy)\, x = y)\, (Sxt \land Kx)$$
$$\exists x\, (Wx \land \forall y\, (Wy \to x = y) \land (Sxt \land Kx))$$

[K: λx (x *was well-known*); S: λxy (x *spoke to* y); W: λx (x *is a winner*); t: *Tom*]

with the description operator:

The winner, who spoke to Tom, was well-known.
The winner spoke to Tom ∧ *the winner was well-known*
S *the winner* Tom ∧ K *the winner*
S(lx x *is a winner*)t ∧ K(lx x *is a winner*)

S(lx Wx)t ∧ K(lx Wx)

d. *using Russell's analysis:*

Every number greater than one is greater than its positive square root
(∀x: x *is a number greater than one*) x *is greater than its positive square root*
(∀x: x *is a number* ∧ x *is greater than one*) x *is greater than the positive square root of* x
(∀x: Nx ∧ Gxo) *the positive square root of* x *is such that (*x *is greater than it)*
(∀x: Nx ∧ Gxo) (∃y: y *is a positive square root of* x ∧ *only* y *is a positive square root of* x) x *is greater than* y
(∀x: Nx ∧ Gxo) (∃y: (y *is positive* ∧ y *is a square root of* x) ∧ (∀z: ¬ z = y) ¬ (z *is positive* ∧ z *is a square root of* x)) Gxy

(∀x: Nx ∧ Gxo) (∃y: (Py ∧ Syx) ∧ (∀z: ¬ z = y) ¬ (Pz ∧ Szx)) Gxy
∀x ((Nx ∧ Gxo) → ∃y ((Py ∧ Syx) ∧ ∀z (¬ z = y → ¬ (Pz ∧ Szx)) ∧ Gxy))
or:
(∀x: Nx ∧ Gxo) (∃y: (Py ∧ Syx) ∧ (∀z: Pz ∧ Szx) y = z) Gxy
∀x ((Nx ∧ Gxo) → ∃y ((Py ∧ Syx) ∧ ∀z ((Pz ∧ Szx) → y = z) ∧ Gxy))

[G: λx (x *is greater than* y); N: λx (x *is a number*); P: λx (x *is positive*); S: λxy (x *is a square root of* y)]

with the description operator:

Every number greater than one is greater than its positive square root
(∀x: x *is a number* ∧ x *is greater than one*) x *is greater than* *the positive square root of* x
(∀x: Nx ∧ Gxo) G x *the positive square root of* x
(∀x: Nx ∧ Gxo) Gx(ly y *is a positive square root of* x)
(∀x: Nx ∧ Gxo) Gx(ly (y *is a positive* ∧ y *is a square root of* x))

(∀x: Nx ∧ Gxo) Gx(ly (Py ∧ Syx))
∀x ((Nx ∧ Gxo) → Gx(ly (Py ∧ Syx)))

2. a. (∃x: x *owns Spot* ∧ (∀y: ¬ y = x) ¬ y *owns Spot*) x *called*
(∃x: x *owns Spot* ∧ *only* x *owns Spot*) x *called*
The owner of Spot is such that (it called)

Spot's owner called

b. *John found* (lx (x *is a photographer* ∧ x *enlarged* (ly y *is a picture of John*)))
John found (lx (x *is a photographer* ∧ x *enlarged the picture of John*))
John found (lx (x *is a photographer who enlarged the picture of John*))

John found the photographer who enlarged the picture of him

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