Phi 270 F02 test 1

1. Define entailment for the special case of three premises by completing the following: φ, ψ, χ ⇒ θ if and only if ... . (Your answer need not replicate the wording of the text's definitions, but it should define entailment in terms of truth values and possible worlds.)
   [answer]

2. Suppose that each of φ and ψ is a tautology (i.e., ⇒ φ and also ⇒ ψ). What, if anything can you conclude about the equivalence of φ and ψ? That is, do you have enough information to conclude that they are equivalent (i.e., that φ ⇔ ψ)? to conclude that they aren't equivalent?--or can't you say for sure? Explain your answer by reference to the definitions of tautologousness and equivalence, using the concepts of possible worlds and truth values that appear in those definitions.
   [answer]

3. Does a statement always entail everything it implicates? If you answer yes, explain why using the definitions of entailment and implicature? If you answer no, give an example of a case of implicature that isn't a case of entailment.
   [answer]

4. Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation. Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

   Jack saw the book and told Jill about it, but she had already read it

   [answer]

Use derivations to check whether each of the entailments below holds. If one fails, present a counterexample--that is, provide a table in which you calculate the truth values of the premises and conclusion on an assignment of truth values which divides an open gap.

5. (A ∧ B) ∧ (C ∧ D) ⇒ A ∧ D
   [answer]

6. A ∧ D ⇒ (A ∧ B) ∧ (C ∧ D)
   [answer]

7. [This question was on a topic not covered in F04] Use replacement principles to put the following sentence into list normal form (i.e., to transform it into a sentence in which no conjunction is the left component of a conjunction and letters appear in alphabetical order without repetition):

   ((A ∧ B) ∧ C) ∧ A

   [answer]

8. Why is single dead-end open gap enough to show that an entailment does not hold? That is, why must all gaps close for a derivation to show that an entailment *does* hold?
[answer]

Phi 270 F02 test 1 answers

1. φ, ψ, χ ⇒ θ if and only if there is no possible world in which φ, ψ, and χ are true but θ is false

2. φ and ψ are equivalent. Because each is a tautology, there is no possible world in which either is false, so there is no possible world in which they have different truth values.

3. No. Any true sentence with a false implicature provides an example of an implicature that isn't an entailment because no true sentence can entail a false one, so you need only provide an example of a true sentence with a false implicature.

4. *Jack saw the book and told Jill about it, but she had already read it*

Jack saw the book and told Jill about it ∧ *Jill had already read the book*

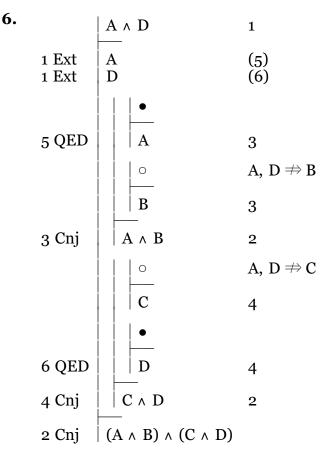(*Jack saw the book* ∧ *Jack told Jill about the book*) ∧ *Jill had already read the book*

$$(S \land T) \land R$$

both both S and T and R

[R: *Jill had already read the book*; S: *Jack saw the book*; T: *Jack told Jill about the book*]

5.

	(A ∧ B) ∧ (C ∧ D)	1
1 Ext	A ∧ B	2
1 Ext	C ∧ D	3
2 Ext	A	(5)
2 Ext	B	
3 Ext	C	
3 Ext	D	(6)
	•	
5 QED	A	4
	•	
6 QED	D	4
4 Cnj	A ∧ D	

6.

	A ∧ D	1
1 Ext	A	(5)
1 Ext	D	(6)

5 QED   ●   A   3

A, D ⇏ B

B   3

3 Cnj   A ∧ B   2

A, D ⇏ C

C   4

6 QED   ●   D   4

4 Cnj   C ∧ D   2

2 Cnj   (A ∧ B) ∧ (C ∧ D)

This derivation could be ended after the first dead-end open gap
appears at stage 3. Any one of the three interpretations below is
enough to present as a counterexample (though the third serves as
a counterexample only once the second dead-end open gap is
reached).

A B C D	A ∧ D	/	(A ∧ B)	∧	(C ∧ D)
T F T T	Ⓣ		F	Ⓕ	T
T F F T	Ⓣ		F	Ⓕ	F
T T F T	Ⓣ		T	Ⓕ	F

7. [This question was on a topic not covered in F04]

$$((A \wedge B) \wedge C) \wedge A$$
$$\Leftrightarrow$$
$$A \wedge ((A \wedge B) \wedge C)$$
$$\Leftrightarrow$$
$$A \wedge (A \wedge (B \wedge C))$$
$$\Leftrightarrow$$
$$(A \wedge A) \wedge (B \wedge C)$$
$$\Leftrightarrow$$
$$A \wedge (B \wedge C)$$

8. A dead-end open gap is always divided by an interpretation that
also makes the initial resources of the derivation true and its initial

goal false. So any dead-end open gap shows that the derivation for which the derivation is constructed can have true premises and a false conclusion.