

**Phi 270 Foo test 4** in pdf format

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *Notice the special instructions for 2.*

1. *Only necessary projects were funded.* [Different interpretations of the scope of *only* are possible here; any of them will do.]  
[ answer ]
2. *Tom can solve the puzzle if anyone can.* [In this case, restate your answer using an unrestricted quantifier.]  
[ answer ]
3. *No one received every vote*  
[ answer ]

Use derivations to establish the validity of the following arguments. You may use attachment rules. English interpretations are suggested but remember that they play no role in derivations, and don't hesitate to ignore them if they don't help you think about the derivations.

4. 
$$\begin{array}{l} (\forall x: Dx) Mx \\ \hline (\forall x: \neg Ax) \neg Mx \\ \hline \forall x (Dx \rightarrow Ax) \end{array}$$
  
[ answer ]  
[A:  $\lambda x$  (x is an animal); D:  $\lambda x$  (x is dog); M:  $\lambda x$  (x is a mammal)]

5. 
$$\begin{array}{l} \forall x (\forall y: Py \wedge Byx) Dyx \\ \hline (\forall x: Px) (\forall y: Bxy) Dxy \end{array}$$
  
[ answer ]

*Everyone who has built anything is proud of it / Everyone is proud of everything he or she has built*

6. Use a derivation to show that the following argument is not valid and describe a structure (by using either a diagram or tables) that divides one of the derivation's open gaps.

$$\begin{array}{l} (\forall x: Rxx) \neg Fx \\ \forall x Rxc \\ \hline \forall x (\forall y: Fy) \neg Rxy \end{array}$$
  
[ answer ]

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**Phi 270 Foo test 4 answers**

1. *Only necessary projects were funded*  
 $(\forall x: \neg x \text{ was a necessary project}) \neg x \text{ was funded}$   
 $(\forall x: \neg (x \text{ was a project} \wedge x \text{ was necessary})) \neg x \text{ was funded}$   
 $(\forall x: \neg (Px \wedge Nx)) \neg Fx$   
or:  $(\forall x: Px \wedge \neg Nx) \neg Fx$ —i.e., *No unnecessary projects were funded*;  
or:  $(\forall x: Nx \wedge \neg Px) \neg Fx$ —i.e., *Among the necessities only projects were funded*  
[F:  $\lambda x$  (x was funded); N:  $\lambda x$  (x was necessary); P:  $\lambda x$  (x was a project)]

2. *Tom can solve the puzzle if anyone can*  
 $(\forall x: x \text{ is a person})$  *Tom can solve the puzzle if x can*  
 $(\forall x: Px)$  (Tom can solve the puzzle  $\leftarrow$  x can solve the puzzle)  
 $(\forall x: Px)$  (S Tom the puzzle  $\leftarrow$  S x the puzzle)  
 $(\forall x: Px)$  (Stp  $\leftarrow$  Sxp) [or:  $(\forall x: Px)$  (Sxp  $\rightarrow$  Stp)]  
 $\forall x$  (Px  $\rightarrow$  (Stp  $\leftarrow$  Sxp)) [or:  $\forall x$  (Px  $\rightarrow$  (Sxp  $\rightarrow$  Stp))]  
[P:  $\lambda x$  (x is a person); S:  $\lambda xy$  (x can solve y); p: *the puzzle*; t: *Tom*]

3. *No one received every vote*  
 $(\forall x: x \text{ is a person})$   $\neg$  x *received every vote*  
 $(\forall x: Px)$   $\neg$  x *received every vote*  
 $(\forall x: Px)$   $\neg$   $(\forall y: y \text{ is a vote})$  x *received y*  
 $(\forall x: Px)$   $\neg$   $(\forall y: Vy)$  Rxy  
[P:  $\lambda x$  (x is a person); R:  $\lambda xy$  (x received y); V:  $\lambda x$  (x is a vote)]

*Incorrect answers:*

- $(\forall x: Px)$   $(\forall y: Vy)$   $\neg$  Rxy says *No one received any vote*  
 $\neg$   $(\forall x: Px)$   $(\forall y: Vy)$  Rxy says *Not everyone received every vote*  
 $(\forall y: Vy)$   $\neg$   $(\forall x: Px)$  Rxy says *No vote is such that everyone received it*

- 4.
- |       |                                   |     |
|-------|-----------------------------------|-----|
|       | $(\forall x: Dx) Mx$              | a:3 |
|       | $(\forall x: \neg Ax) \neg Mx$    | a:4 |
|       | (a)                               |     |
|       | Da                                | (3) |
| 3 SB  | Ma                                | (4) |
| 4 SC  | Aa                                | (5) |
|       | •                                 |     |
|       | Aa                                | 2   |
| 5 QED |                                   |     |
|       | Da $\rightarrow$ Aa               | 1   |
| 2 CP  |                                   |     |
| 1 UG  | $\forall x$ (Dx $\rightarrow$ Ax) |     |

- 5.
- |       |  |         |
|-------|--|---------|
|       | $\forall x$ $(\forall y: Py \wedge Byx) Dyx$ | b:3     |
|       | (a)  |         |
|       | Pa   | (6)     |
|       | (b)  |         |
|       | Bab  | (3),(7) |
| 3 UI  | $(\forall y: Py \wedge Byb) Dyb$             | a:5     |
|       | $\neg$ Dab                                   | (5)     |
| 5 SC  | $\neg$ (Pa $\wedge$ Bab)                     | 6       |
| 6 MPT | $\neg$ Bab                                   | (7)     |
|       | $\perp$                                      | 4       |
| 7 Nc  |  |         |
|       | Dab  | 2       |
| 4 IP  |  |         |
|       | $(\forall y: Bay) Day$                       | 1       |
| 2 RUG |  |         |
| 1 RUG | $(\forall x: Px)$ $(\forall y: Bxy) Dxy$     |         |

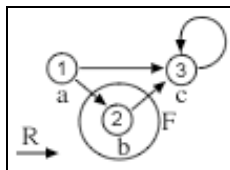
[This can be done without the *reductio* argument begun at stage 4 by using an attachment rule to derive Pa  $\wedge$  Bab in order to exploit  $(\forall y: Py \wedge Byb) Dyb$  for a]

6.

$(\forall x: Rxx) \neg Fx$   
 $\forall x Rxc$

b:3,c:7,a:9  
 a:4,b:5,c:6

	(a)		
	(b)	Fb	(3)
3 SC		$\neg Rbb$	
4 UI		Rac	
5 UI		Rbc	
6 UI		Rcc	(7)
7 SB		$\neg Fc$	
		Rab	
		$\neg Raa$	
		○	Fb, $\neg Rbb$ , Rac, Rbc, Rcc, $\neg Fc$ , Rab, $\neg Raa \Rightarrow \perp$
		$\perp$	10
10 IP		Raa	9
		$\neg Fa$	
		○	Fb, $\neg Rbb$ , Rac, Rbc, Rcc, $\neg Fc$ , Rab, $\neg Fa \Rightarrow \perp$
		$\perp$	9
9 MCR		$\perp$	8
8 RAA		$\neg Rab$	2
2 RUG		$(\forall y: Fy) \neg Ray$	1
1 UG		$\forall x (\forall y: Fy) \neg Rxy$	



divides both open gaps