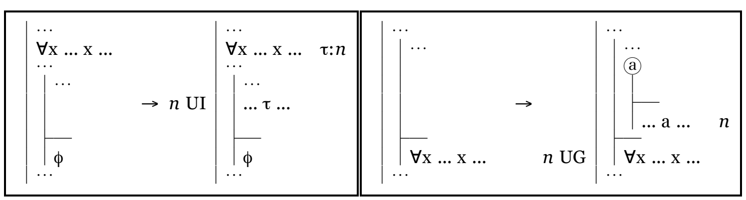
7.5.s. Summary

7.5.1. The universal quantifiers and conjunction may both be used to say that each of a group of claims is true. This overlap in function indicates an analogy between these logical constants that can be seen also in the laws of entailment for them. The analogue to a component of a conjunction is an instance of a universal, in which applies the universal's quantified predicate is predicated a term. A universal is rarely equivalent to an actual conjunction of its instances, but for a given referential range **R**, it behaves like a possibly infinite conjunction of instances in a language enriched by adding the IDs of all values in **R**—i.e., it behaves like the conjunction of its instances in an expansion of the language by **R**. When we do not fix the range **R**, a universal $\forall x\ \theta x$ is not associated with any definite set of instances, but we still know that its instances $\theta\tau$ are all predications of $\lambda x\ \theta x$; and these two features are reflected in the laws of entailment for universals.

7.5.2. In the case of an unrestricted universal, we can state a principle of universal instantiation, which says that the universal implies each of its instances, and we may use this with the law for lemmas to get a law for this sort of universal as a premise. We can describe the role of an unrestricted universal as a conclusion by using the idea of a general argument, in which an instance of a generalization is established in such a way that we may generalize from it to a universal claim. It is sufficient for an argument to be a general one that the term for which the instance is given not be compound, that it not appear in the premises, and that it not appear in the generalization we wish to conclude. Such a term is parametric or a parameter for the argument. The law for the unrestricted conditional as a conclusion then tells us that we can conclude a universal from given premises when we can conclude an instance of it for a parametric term.

7.5.3. In implementing the laws for universals as conclusions, we flag scope lines by terms that are being used as parameters; such terms can appear only to the right of their scope lines. We plan for an unrestricted universal goal by planning to use the rule Universal Generalization (UG). It directs us to set up a flagged scope line with an instance for the parameter as a new goal. While we introduce new terms when planning for universal conclusions, the rule for exploiting universal resources— Universal Instantiation (UI) —should be used only for terms already appearing in the gap—provided there is at least one such term. The exploitation of universals can never be considered complete, and an available universal resource is always an active resource; but exploitation rules do render universals inactive for particular terms and should be applied only to terms for which the universal remains active.



7.5.x. Exercise questions

1. Give the instances of each of the following for the terms a, b, and c (remembering that you will drop the main quantifier, and only the main one, when giving an instance):

a. $\forall x\ Fx$
b. $\forall y\ Fy$
c. $\forall x\ Rxa$
d. $\forall x\ Saxb$
e. $\forall x\ \forall y\ Rxy$
f. $\forall x\ (Fx \to Gx)$
g. $\forall x\ (Fx \to Gd)$
h. $\forall x\ (Fx \to \forall y\ Rxy)$
i. $\forall x\ (Fx \to \forall x\ Rxx)$

2. Use the system of derivations to establish each of the following. You may use detachment and attachment rules.

a. $\forall x\ Fx, \forall x\ (Fx \to Gx) \Rightarrow Ga$
b. $\forall x\ (Fx \wedge Gx) \Rightarrow Fa \wedge Gb$
c. $\forall x\ Rxa, \forall x\ (Rbx \to Gx) \Rightarrow Ga$
d. $\forall x\ Fx, \forall x\ (Fx \to Gx) \Rightarrow \forall x\ Gx$
e. $\forall x\ (Fx \wedge Gx) \Leftrightarrow \forall x\ Fx \wedge \forall x\ Gx$
f. $\forall x\ \forall y\ Rxy \Rightarrow (Rab \wedge Rbb) \wedge Rca$
g. $\forall x\ \forall y\ Rxy \Rightarrow \forall y\ Rya$
h. $\forall x\ \forall y\ (Rxy \to \neg Ryx) \Rightarrow \forall x\ \neg Rxx$
i. $\forall x\ \forall y\ \forall z\ ((Rxy \wedge Ryz) \to Rxz), \forall x\ \neg Rxx \Rightarrow \forall x\ \forall y\ (Rxy \to \neg Ryx)$

Homework assigned Wed 11/10 and due Fri 11/12

(i) Use derivations to show: $\forall x\ Fx, \forall x\ \forall y\ Rxy \Rightarrow \forall y\ (Fy \wedge \forall x\ Rxy)$

(ii) Analyze: *No one in the audience applauded every performer*