7.2.s. Summary

7.2.1. Generalizations will be expressed symbolically using quantifiers, operations that take predicates as input and yield sentences as output. More specifically, we will use two universal quantifiers both written using the symbol ∀ (for all). The sentences form using these quantifiers will be called universals. The two quantifiers are the restricted universal quantifier, which applies to a pair of predicates to form a sentence, and the unrestricted universal quantifier, which applies to a single predicate. We will apply quantifiers only to abstracts. Since any pair of abstracts can be written in the form λx (...x...) and λx (---x---) using the same variable, we can abbreviate universal sentences as (∀x:...x...) ---x--- and ∀x ---x---. These may be put into English notation as `Everything`, x, `such that` ρx `is such that` θx `and` `Everything`, x, `is such that` θx. (Here the word *thing* is used as a dummy restriction that merely provides a hook for the relative clause.) The component expressions ...x... and ---x---, the restricting and quantified formulas of the universal, will not ordinarily be sentences in the strictest sense because they will contain free occurrences of the variable x. (The exceptions are the bodies of vacuous abstracts expressing predicates with a constant value.) Such expressions are included in the broader class of formulas, among which sentences are distinguished as closed formulas. Terms, too, can be classified as open or closed. A restricted universal says that the extension of the first predicate to which it is applied, the restricting predicate, is included in the extension of the second, the quantified predicate—i.e., it says that the second expresses a property that is at least as general as that expressed by the first. The unrestricted quantifier says that the quantified predicate to which it applies is universal, that it is a predicate that expresses a fully general property. An unrestricted universal sentence can be restated as a restricted universal whose domain predicate is universal, and a restricted universal can be restated as an unrestricted universal provided we make the attribute predicate conditional on the domain predicate.

7.2.2. An English generalization may be analyzed symbolically by using restricting and quantified predicates that capture its domain and attribute. If its domain consists of all reference values, an unrestricted universal may be used and we need only capture its attribute. In an affirmative generalization, the attribute predicate will be the quantified predicate of the English generalization while in a negative generalization it will be the negation of the quantified predicate. A formula applying the restricting predicate can be formed from the class indicator C by using the form x *is a* C, adding negation if the generalization is complementary. (However, we start with x *is a person* in the case of *everyone* and *no one*.) The phrase *all and only* is used to express a conjunction of affirmative direct and negative complementary generalizations; but a generalization of this sort can be analyzed also by an unrestricted universal applying to a biconditional predicate because the two generalizations it implies can be expressed using an *if*-conditional and an *only-if*-conditional, respectively.

7.2.3. Bounds and exceptions may be captured by conjoining to the restricting predicate a predicate or predicates of the same form, negated in the case of exceptions.

7.2.x. Exercise questions

1. Restate, with unrestricted quantifiers, the generalizations below that employ restricted quantifiers—and vice versa. Write out English readings for the results.

a. (∀x: Fx) Gx
b. ∀x (Fx → ¬ Gx)
c. (∀x: Fx ∧ ¬ Gx) Hx
d. ∀x ((Px ∧ ¬ Rxx) → Rxa)
e. (∀x: Rxa ∧ ¬ Rbx) ¬ (Fx ∨ Gx)
f. ∀x ((Fx ∨ Gx) → (Hx ∧ ¬ Kx))

2. Analyze the following in as much detail as possible, stating the resulting form using both restricted and unrestricted quantifiers:

a. *Everyone had heard about the accident*.
b. *Every relative of Sam agreed with him about the issue*.
c. *Edna took pleasure in none of her possessions*.
d. *Tom found only empty boxes*
e. *The survey was sent to all members of the organization except its officers*.
f. *Only countries bordering the Pacific will prosper*.

3. State in idiomatic English the generalizations that could be represented symbolically by the following:

a. (∀x: x *is a dog*) x *chases cats*.
b. (∀x: x *is a hole*) *Holly patched* x.
c. (∀x: x *is a person*) ¬ x *volunteered*.
d. (∀x: ¬ x *is a cockroach*) ¬ x *will survive*.
e. ∀x ¬ x *seemed right*.
f. (∀x: x *was a reviewer* ∧ ¬ x *was a friend of the director*) x *panned the movie*.
g. (∀x: x *is a bird* ∧ ¬ x *is an early bird*) ¬ x *gets a worm*.
h. (∀x: ¬ x *is a small child*) *the movie bored* x.

Homework assigned Wed 11/3 and due Fri 11/5

(i) Analyze and restate the result using unrestricted quantifiers:

Dan checked every pocket, but he had only coins

(ii) Synthesize an English sentence that has the following analysis:

(∀x: Ix) ¬ Sxc
[I: λx (x is an inspector); S: λxy (x saw y); c: the crack]