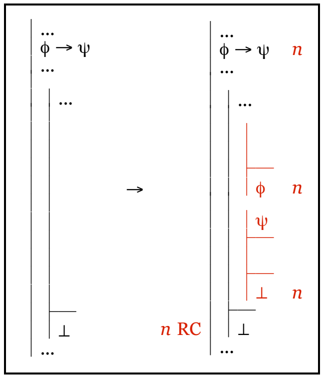
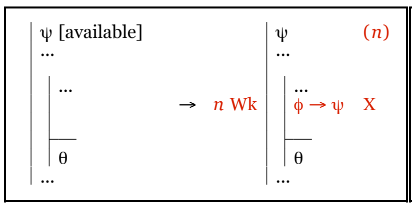
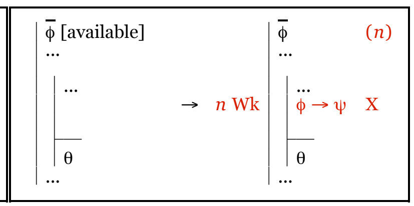
5.4.s. Summary

5.4.1. The law for the conditional as a premise applies only to *reductio* arguments and provides a way of rejecting a conditional by deriving its antecedent ϕ from the premises and reducing its consequent to absurdity given the premises. The corresponding derivation rule is Rejecting a Conditional (RC).



5.4.2. This rule reflects the fact that a conditional is false when its antecedent is true and its consequent is false. The rules of Weakening (Wk) that have conditionals as conclusions reflect the fact that a conditional is true if its consequent is and also if its antecedent is false.





With these rules, the system of derivations for truth-functional logic is complete.

Rules for developing gaps		
	for resources	*for goals*
atomic sentence		IP
negation ¬ ϕ	CR (if ϕ is not atomic and the goal is ⊥)	RAA
conjunction ϕ ∧ ψ	Ext	Cnj
disjunction ϕ ∨ ψ	PC	PE
conditional ϕ → ψ	RC (if the goal is ⊥)	CP

Detachment rules (optional)		
main resource	*auxiliary resource*	*rule*
ϕ → ψ	ϕ	MPP
ϕ → ψ	ψ̄	MTT
ϕ ∨ ψ	ϕ̄ or ψ̄	MTP
¬ (ϕ ∧ ψ)	ϕ or ψ	MPT

Rules for closing gaps		
when to close	*rule*	
the goal is also a resource	QED	
sentences ϕ and ¬ ϕ are resources & the goal is ⊥	Nc	
⊤ is the goal	ENV	
⊥ is a resource	EFQ	*Basic system*

Attachment rules		
added resource	*rule*	
ϕ ∧ ψ	Adj	*Added rules (optional)*
ϕ → ψ	Wk	
ϕ ∨ ψ	Wk	
¬ (ϕ ∧ ψ)	Wk	
Rule for lemmas		
prerequisite	*rule*	
the goal is ⊥	LFR	

5.4.x. Exercise questions

1. Use derivations to check each of the claims below; if a derivation indicates that a claim fails, present a counterexample that divides an open gap. Since **d** is a claim of tautologousness, it is established by a derivation that begins with only a goal and no initial premises.

- **a.** A → B ⇔ ¬ A ∨ B
- **b.** (A ∧ B) → C ⇔ A → C
- **c.** (A → B) ∧ (B → C) ⇔ A → C
- **d.** ⇒ ((A → B) → A) → A

2. Construct derivations for each of the following. These exercises are designed to make attachment rules often useful. The derivations can be constructed for the English sentences in **e-g** without first analyzing them since you generally need to recognize only the main connective and the immediate connectives in order to know what rules apply; however, the abbreviated notation provided by an analysis may be more convenient.

- **a.** (A ∧ B) → C, (C ∨ D) → E, A, B ⇒ E
- **b.** (A ∨ ¬ B) → C ⇒ ¬ C → B
- **c.** ¬ (A ∧ B), B ∨ C, D → ¬ C ⇒ A → ¬ D
- **d.** C → ¬ (A ∨ B), E ∨ ¬ (D ∧ ¬ C), D ⇒ A → E
- **e.** *Tom will go through Chicago and visit Sue*
  Tom won't go through both Chicago and Indianapolis
  Tom won't visit Ursula without going through Indianapolis
  ———
  Tom will visit Sue but not Ursula
- **f.** *Either we spend a bundle on television or we won't have wide public exposure*
  If we spend a bundle on television, we'll go into debt
  Either we have wide public exposure or our contributions will dry up
  We'll go into debt if our contributions dry up and we don't have large reserves
  We won't have large reserves
  ———
  We'll go into debt
- **g.** *If Adams supports the plan, it will go though provided Brown doesn't oppose it*
  Brown won't oppose the plan if either Collins or Davis supports it
  ———
  The plan will go through if both Adams and Davis support it

Homework assigned Mon 10/18 and due Wed 10/20

Use derivations to check the following, checking it twice, once without using attachment rules and once using them:

(A → B) → ¬ (C → D) ⇒ B → ¬ D