3.4.s. Summary

3.4.1. We again establish the adequacy of our system by showing that it is sufficient, conservative, and decisive. The arguments for sufficiency and decisiveness turn on new features that appeared with negation. A gap that remains open at a dead end will now always have ⊥ as its goal and its resources are limited to ⊤, atomic sentences, and negated atomic sentences, with no resource being the negation of another. Any such gap can be divided by an interpretation that makes all its active resources true. Also, we can show that our new rules will not lead us on forever by showing that they are progressive by leading us always to replace goals or resources by others of a lower grade eventually leading us to goals and resources that are minimal, a class that includes atomic sentences and their negations.

3.4.2. Dead-end gaps will now have proximate arguments that are *reductio*s, so the failure of a derivation will turn on the failure of a *reductio* and thus on the fact that a certain set of sentences (the premises of the *reductio*) is a consistent set. Thus all examples of failures of entailment rest on examples of consistency.

3.4.x. Exercise questions

1. The following arguments are not formally valid. In each case, use a derivation to show this and present a counterexample that the derivation leads you to.

- **a.** ¬ B / ¬ (A ∧ ¬ B)
- **b.** ¬ (A ∧ B) / ¬ A ∧ ¬ B
- **c.** ¬ (A ∧ B), ¬ (B ∧ C) / ¬ (A ∧ C)

2. Use derivations to check the following claims of entailment. If the claim fails present a counterexample that it leads you to.

- **a.** ¬ (A ∧ ¬ B) ⇒ B
- **b.** ¬ (A ∧ B) ⇒ ¬ (B ∧ A)
- **c.** ¬ (A ∧ ¬ B) ⇒ ¬ (B ∧ ¬ A)
- **d.** ¬ (A ∧ B), ¬ (B ∧ C), B ⇒ ¬ A ∧ ¬ C
- **e.** ¬ (A ∧ ¬ (B ∧ ¬ (C ∧ ¬ D))) ⇒ ¬ (A ∧ ¬ (B ∧ D))

Homework assigned Mon 9/27 and due Wed 9/29

Use a derivation to show the following argument is not formally valid and present a counterexample that divides a dead-end gap: ¬ (¬ B ∧ C), ¬ ¬ A ⇒ A ∧ B