3.1.s. Summary

3.1.1. Negation is an operation associated with the English word *not*. It generates a compound sentence from a single component, so it is a connective that serves to modify a sentence rather than to combine sentences. The not symbol ¬ is our notation for negation. As English notation for ¬ ϕ, we use `not` ϕ.

3.1.2. A sentence and its negation cannot be both true (they are mutually exclusive) and cannot be both false (they are jointly exhaustive); in short, they must have different truth values (they are contradictory). Each leaves open the possibilities the other rules out and rules out the possibilities the other leaves open. This means that negation, like conjunction, has a truth table; in other words it is a truth-functional connective. Not all connectives are truth-functional. Truth-functional logic is the branch of logic which studies those that are, but there are branches of logic—such as tense logic and modal logic—in which non-truth-functional connectives are studied.

3.1.3. Negation appears in English not only in connection with the word *not* but also with negative prefixes (though such a prefix does not always mark negation because it does not always produce a sentence that is contradictory to the original). Negation also appears with uses of *no* in phrases of the form *no X*, uses that can often be treated as the negation of *at least one* or *some*. The same sort of treatment is usually what is required when *not* appears along with the word *any* (which usually must be rephrased when *not* is removed). By negating a negation, we can produce a double negation, but this undoes the negation rather than generating a logical form with new properties.

3.1.4. The really new ideas come with the negation of conjunctions, but conjunctions whose components may involve negation also provide important forms of expression. A number of forms are shown below, with labels that suggest the sort of English sentences they serve to analyze:

not-both form	¬ (ϕ ∧ ψ)	`not both` ϕ `and` ψ
not-but form	¬ ϕ ∧ ψ	`both not` ϕ `and` ψ
but-not form	ϕ ∧ ¬ ψ	`both` ϕ `and not` ψ
not-and-not form	¬ ϕ ∧ ¬ ψ	`both not` ϕ `and not` ψ
not-without form	¬ (ϕ ∧ ¬ ψ)	`not both` ϕ `and not` ψ

That the last is the denial of the third reflects the fact that *without* can be used to express a *but-not* form. Also *neither-nor* can be used to express a *not-and-not* form. More generally, negation and conjunction form a truth-functionally complete set of connectives in the sense that any truth-functional compound can be expressed using them alone.

3.1.x. Exercise questions

1. Analyze each of the following sentences in as much detail as possible.

- **a.** *The soup was hot but not too hot, and thick but not too thick.*
- **b.** *The equipment isn't here and it's unlikely to arrive soon.*
- **c.** *No one answered the phone even though it rang 10 times.*
- **d.** *The alarm must have gone off, but Ted didn't hear anything.*
- **e.** *They won't both meet the deadline and stay within the budget.*
- **f.** *They won't meet the deadline, but they will stay within the budget.*
- **g.** *They won't meet the deadline, and they won't stay within the budget.*
- **h.** *Tod shut off the alarm without waking up.*
- **i.** *They won't meet the deadline without going over the budget.*
- **j.** *Larry joined in, but not without being coaxed.*
- **k.** *Ann liked the movie, but neither Bill nor Carol did.*

2. Restate each of the forms below, putting English notation into symbols and vice versa. Indicate the scope of connectives in the result by underlining.

- **a.** ¬ ¬ (A ∧ B)
- **b.** ¬ (¬ A ∧ B)
- **c.** `both not` A `and both not` B `and` C
- **d.** `both not both` A `and` B `and not` C

3. Synthesize idiomatic English sentences that express the propositions associated with the logical forms below by the intensional interpretations that follow them.

- **a.** C ∧ ¬ F
  [C: *it was cold*; F: *there was frost*]
- **b.** ¬ S ∧ (H ∧ I)
  [H: *Sue heard a crash*; I: *Sue went to investigate*; S: *someone saw the accident*]
- **c.** (D ∧ N) ∧ ¬ P
  [D: *it was a design*; N: *it was new*; P: *it pleased someone*]
- **d.** ¬ (I ∧ N)
  [I: *we'll win in Iowa*; N: *we'll win in New York*]
- **e.** ¬ I ∧ N
  [I: *we'll win in Iowa*; N: *we'll win in New York*]
- **f.** ¬ (I ∧ ¬ L)
  [I: *we'll win in Iowa*; L: *we'll lose in New York*]

4. Calculate truth values for all components of the forms below using the extensional interpretation provided in each case.

- **a.**

A	B	C	A ∧ ¬ (B ∧ C)
T	F	F	

- **b.**

A	B	C	A ∧ (¬ B ∧ C)
T	F	F	

- **c.**

A	B	C	D	(¬ A ∧ ¬ B) ∧ (¬ (A ∧ C) ∧ D)
F	T	T	T	

Homework assigned Fri 9/17 and due Wed 9/22

Analyze in as much detail as possible, and state the result using both symbolic and English notation:

Ann and Bill didn't both like the food, and neither ate there again

(Notice that *Ann and Bill didn't both like the food* does not mean the same thing as *Ann and Bill both didn't like the food*.)