8.6.1. The role of definite descriptions in entailment

If Russell’s analysis of definite descriptions is accepted, their logical properties follow from those of the logical constants used in the analysis, but the description operator is a new symbol and studying its logical properties requires stating new principles for it. Studying the logical properties of the description operator is part of what must be done to decide on the correct analysis of English, and these properties were already discussed informally in 8.4.2.

To give a more explicit account of them, we must first find a place for the description operator in our semantic scheme. All our logical constants so far—whether the connectives, the quantifiers, or the identity predicate—have been ways of producing compound formulas. The description operator, on the other hand, yields a compound term when it is applied to a predicate. This means that the extension of ι will be a function from the extensions of one-place predicates to reference values. We can represent the extension of a one-place predicate by the set of reference values of which it is true, so the extension of the description operator can be seen as a function which takes sets of reference values as input an yields single reference values as output.

We have required that a term $\iota x\ \rho x$ formed using the description operator refer to the single value in the extension of ρ if there is just one value and that it refer to the nil value otherwise. This means that the extension of the description operator is not settled until we identify the nil value as a specific value in the referential range. This identification must be considered a further component of a structure, a respect in which two structures may differ. So when we make the description operator a part of our language, we require that a structure distinguish a member of the referential range as the nil value. This will serve as the reference value of the constant individual term $*$ introduced in 8.4.2. Then, to find the semantic value given to $\iota x\ \rho x$ by a structure, we find the extension the structure gives to the predicate ρ. If the extension of ρ has just one member, that reference value will be the extension of $\iota x\ \rho x$; otherwise, the extension of $\iota x\ \rho x$ is the value the structure assigns to $*$.

A specification made regarding structures and the interpretation of logical vocabulary will typically result in some logical law. For example, the requirement that the referential range serve both as a source of extensions for terms and as the domain of unrestricted universals gives us the principle of universal instantiation. And even the simple requirement that a referential range be non-empty yields the law $\forall x\ \theta x$

$\Rightarrow \exists x\, \theta x$, which assures us that universal predicates are exemplified. In the case of our specifications for definite descriptions and the nil value, we get what we will refer to as the ***law for descriptions***. In the case of a definite description $\imath x\, \rho x$ with no free variables, this takes the form:

$$\Rightarrow (\exists z{:}\ \rho z \wedge (\forall y{:}\ \rho y)\ z = y)\ \imath x\, \rho x = z \vee ((\forall x{:}\ \rho x)\ (\exists y{:}\ \rho y)\ \neg\, x = y \wedge \imath x\, \rho x = *).$$

The existential quantifier in the first disjunct should be familiar from Russell's analysis of definite descriptions. The whole first disjunct might be read as *Something such that* ρ *fits it and it is all that* ρ *fits is such that the thing* ρ *fits is it* or, a little more idiomatically, as *The thing* ρ *fits is something that is all that* ρ *fits*. The second disjunct of the law is a conjunction whose first conjunct says *Anything that* ρ *fits is such that something* ρ *fits is different from it*. This is a compact but somewhat roundabout way of saying that the extension of ρ does not have exactly one member—i.e., if we can find anything in it, we can find something else in it, too. Finally, the last component of the law can be read as *The thing* ρ *fits is the nil*. Putting this all together, the law amounts to the following:

> *Either (i)* ιx ρx *refers to something that is all that* ρ *fits, or (ii)* ρ *does not fit exactly one thing and* ιx ρx *refers to the nil*

The first disjunct specifies the reference of the definite description when this is determined by the description, and the second disjunct specifies the reference when the description does not succeed in determining it.

In 8.4.2 the content of an analysis using the description operator was expressed using a similar disjunction. On that account, a sentence $\theta(\imath x\, \rho x)$ says that either ρ is true of exactly one thing and $(\exists x{:}\ \rho x)\ \theta x$ or ρ is not true of exactly one thing and $\theta *$. Given the law for descriptions, the properties of identity will tell us that

$$\theta(\imath x\, \rho x) \Leftrightarrow (\exists z{:}\ \rho z \wedge (\forall y{:}\ \rho y)\ z = y)\ \theta z \vee ((\forall x{:}\ \rho x)\ (\exists y{:}\ \rho y)\ \neg\, x = y \wedge \theta *)$$

and the right-hand side is a more formal version of the disjunction used in 8.4.2.