

8.5.xa. Exercise answers

1. Some of the derivations below are given in two forms, one that does not use EG and REG and another that does. It is also possible to use the rules REP and REC along with the rules for unrestricted existentials. Such answers are not shown but they can be constructed from the answers that are given by using the substitutions of rules shown in the following table:

rule	alternative approach using REP and REC
PRCh	REP, PCh, Ext
RNcP	REC, NcP (with later uses SB, SC, and MCR replaced by UI and either MPT or CR)
REG	Adj, EG, REC

a.

	$\exists x Fx$ 1 $\forall x (Fx \rightarrow Gx)$ a:2		$\exists x Fx$ 1 $\forall x (Fx \rightarrow Gx)$ a:2	
	<div style="border-left: 1px solid black; padding-left: 5px;"> (a) </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> (a) </div>	
	<div style="border-left: 1px solid black; padding-left: 5px;"> Fa (3) </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> Fa (3) </div>	
2 UI	<div style="border-left: 1px solid black; padding-left: 5px;"> $Fa \rightarrow Ga$ 3 </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> $Fa \rightarrow Ga$ 3 </div>	
3 MPP	<div style="border-left: 1px solid black; padding-left: 5px;"> Ga </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> Ga (4) </div>	
	<div style="border-left: 1px solid black; padding-left: 5px;"> $\forall x \neg Gx$ a:5 </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> $\exists x Gx$ X, (5) </div>	
	<div style="border-left: 1px solid black; padding-left: 5px;"> $\neg Ga$ (6) </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> \bullet </div>	
5 UI	<div style="border-left: 1px solid black; padding-left: 5px;"> \bullet </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> $\exists x Gx$ 1 </div>	
6 Nc	<div style="border-left: 1px solid black; padding-left: 5px;"> \perp </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> $\exists x Gx$ </div>	
4 NcP	<div style="border-left: 1px solid black; padding-left: 5px;"> $\exists x Gx$ 1 </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> $\exists x Gx$ </div>	
1 PCh	$\exists x Gx$		$\exists x Gx$	

b.

	$(\exists x: Fx) Gx$ 1 $(\forall x: Gx) Hx$ a:2		$(\exists x: Fx) Gx$ 1 $(\forall x: Gx) Hx$ a:2	
	<div style="border-left: 1px solid black; padding-left: 5px;"> (a) </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> (a) </div>	
	<div style="border-left: 1px solid black; padding-left: 5px;"> Fa (4) </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> Fa (3) </div>	
	<div style="border-left: 1px solid black; padding-left: 5px;"> Ga (2) </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> Ga (2) </div>	
2 SB	<div style="border-left: 1px solid black; padding-left: 5px;"> Ha (5) </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> Ha (3) </div>	
	<div style="border-left: 1px solid black; padding-left: 5px;"> $(\forall x: Fx) \neg Hx$ a:4 </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> $(\exists x: Fx) Hx$ X, (4) </div>	
4 SB	<div style="border-left: 1px solid black; padding-left: 5px;"> $\neg Ha$ (5) </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> \bullet </div>	
	<div style="border-left: 1px solid black; padding-left: 5px;"> \perp </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> $(\exists x: Fx) Hx$ 1 </div>	
5 Nc	<div style="border-left: 1px solid black; padding-left: 5px;"> \perp </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> $(\exists x: Fx) Hx$ </div>	
3 RNcP	<div style="border-left: 1px solid black; padding-left: 5px;"> $(\exists x: Fx) Hx$ 1 </div>		<div style="border-left: 1px solid black; padding-left: 5px;"> $(\exists x: Fx) Hx$ </div>	
1 PRCh	$(\exists x: Fx) Hx$		$(\exists x: Fx) Hx$	

c.

	$\forall x (Fx \rightarrow Ga)$	b:3
	$\exists x Fx$	2
	(b)	
	Fb	(4)
3 UI	$Fb \rightarrow Ga$	4
4 MPP	Ga	(5)
	•	
5 QED	Ga	2
2 PCh	Ga	1
1 CP	$\exists x Fx \rightarrow Ga$	

	$\exists x Fx \rightarrow Ga$	4
	(b)	
	Fb	(8)
	$\neg Ga$	(4)
4 MTT	$\neg \exists x Fx$	5
	$\forall x \neg Fx$	b:7
7 UI	$\neg Fb$	(8)
	•	
8 Nc	\perp	6
6 NcP	$\exists x Fx$	5
5 CR	\perp	3
3 IP	Ga	2
2 CP	$Fb \rightarrow Ga$	1
1 UG	$\forall x (Fx \rightarrow Ga)$	

	$\exists x Fx \rightarrow Ga$	4
	(b)	
	Fb	(3)
	$\exists x Fx$	X, (4)
3 EG	Ga	(5)
4 MPP	•	
5 QED	Ga	2
2 CP	$Fb \rightarrow Ga$	1
1 UG	$\forall x (Fx \rightarrow Ga)$	

d.

	Fa	(2)
	$(\forall x: x = a) \neg Fx$	a:2
2 SC	$\neg a = a$	(3)
	•	
3 DC	\perp	1
1 RNcP	$(\exists x: x = a) Fx$	

	Fa	(2)
1 EC	$a = a$	(2)
2 REG	$(\exists x: x = a) Fx$	(3)
	•	
3 QED	$(\exists x: x = a) Fx$	

	$(\exists x: x = a) Fx$	1
	(b)	
	$b = a$	a—b
	Fb	(2)
	•	
2 QED=	Fa	1
1 PRCh	Fa	

e.

	$(\exists x: Fx) \forall y Rxy$	2		$(\exists x: Fx) \forall y Rxy$	2
	(a)			(a)	
	(b)			(b)	
	Fb	(4)		Fb	(4)
	$\forall y Rby$	b:5		$\forall y Rby$	b:3
	(4)			Rba	(4)
	$(\forall y: Fy) \neg Rya$	b:4	3 UI	$(\exists y: Fy) Rya$	X, (5)
	(6)		4 REG	•	
4 SB	$\neg Rba$	(6)		(4)	
5 UI	Rba	(6)		(4)	
	•		5 QED	(4)	
6 Nc	\perp	3	2 PRCh	(4)	
3 RNcP	$(\exists y: Fy) Rya$	2	1 UG	(4)	
2 PRCh	$(\exists y: Fy) Rya$	1		(4)	
1 UG	$\forall x (\exists y: Fy) Ryx$			(4)	
	$\forall x (\exists y: Fy) Ryx$			(4)	

f.

	$(\exists x: Gx) Fx$	1
	$\neg Fa$	(4)
	(b)	
	Gb	(3)
	Fb	(4)
	(3)	
3 SC	$(\forall x: \neg x = a) \neg Gx$	b:3
	(4)	
	b = a	a—b
	•	
4 Nc=	\perp	2
2 RNcP	$(\exists x: \neg x = a) Gx$	1
1 PRCh	$(\exists x: \neg x = a) Gx$	

g.	$\forall x (Fx \rightarrow Ga)$	c:3	
	$\forall x (Ga \rightarrow Fx)$	b:5	
	$\exists x Fx$	2	
	<div style="display: flex; align-items: center;"> <div style="border-right: 1px solid black; padding-right: 5px; margin-right: 5px;">(b)</div> <div style="border-right: 1px solid black; padding-right: 5px; margin-right: 5px;">(c)</div> <div style="padding-left: 5px;">Fc</div> <div style="padding-left: 20px;">(4)</div> </div>		
	3 UI	Fc \rightarrow Ga	4
	4 MPP	Ga	(6)
	5 UI	Ga \rightarrow Fb	6
	6 MPP	Fb	(7)
		•	
	7 QED	Fb	2
	2 PCh	Fb	1
	1 UG	$\forall x Fx$	

h. *Everyone loves everyone who loves someone*
Someone loves someone

	$(\forall x: Px) (\forall y: Py \wedge (\exists z: Pz) Lyz) Lxy$	b:5, a:9
	$(\exists x: Px) (\exists y: Py) Lxy$	3
	<div style="display: flex; align-items: center;"> <div style="border-right: 1px solid black; padding-right: 5px; margin-right: 5px;">(a)</div> <div style="padding-left: 5px;">Pa</div> <div style="padding-left: 20px;">(9)</div> </div>	
	<div style="display: flex; align-items: center;"> <div style="border-right: 1px solid black; padding-right: 5px; margin-right: 5px;">(b)</div> <div style="padding-left: 5px;">Pb</div> <div style="padding-left: 20px;">(5), (11)</div> </div>	
	<div style="display: flex; align-items: center;"> <div style="border-right: 1px solid black; padding-right: 5px; margin-right: 5px;">(c)</div> <div style="padding-left: 5px;">Pc</div> <div style="padding-left: 20px;">(7), (10)</div> </div>	
	$(\exists y: Py) Lcy$	4
	<div style="display: flex; align-items: center;"> <div style="border-right: 1px solid black; padding-right: 5px; margin-right: 5px;">(d)</div> <div style="padding-left: 5px;">Pd</div> <div style="padding-left: 20px;">(6)</div> </div>	
	Lcd	(6)
5 SB	$(\forall y: Py \wedge (\exists z: Pz) Lyz) Lby$	c:8
6 REG	$(\exists z: Pz) Lcz$	X, (7)
7 Adj	Pc \wedge $(\exists z: Pz) Lcz$	X, (8)
8 SB	Lbc	(10)
9 SB	$(\forall y: Py \wedge (\exists z: Pz) Lyz) Lay$	b:12
10 REG	$(\exists z: Pz) Lbz$	X, (11)
11 Adj	Pb \wedge $(\exists z: Pz) Lbz$	X, (12)
12 SB	Lab	(13)
	•	
13 QED	Lab	4
4 PRCh	Lab	3
3 PRCh	Lab	2
2 RUG	$(\forall y: Py) Lay$	1
1 RUG	$(\forall x: Px) (\forall y: Py) Lxy$	

Everyone loves everyone

Note that stages 4 and 6 serve only to move us from $(\exists y: Py) Lcy$ to $(\exists z: Pz) Lcz$ —i.e., to change a bound variable. If sentences that differ only in the choice of a letter for a bound variable are regarded as the same, $(\exists y: Py) Lcy$ could be used as a premise for Adj at stage 7 and the use of PRCh at stage 4 would not be needed.

i. *Something is such that nothing other than it is done*

[When *nothing* is analyzed using a negative generalization, a derivation like that below but without stages 6 and 7 could be used.]

	$\exists x \neg (\exists y: \neg y = x) Dy$	2
	$\exists x (\exists y: \neg y = x) (Dx \wedge Dy)$	3
	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; padding-left: 5px; margin-right: 5px;"> $\neg (\exists y: \neg y = a) Dy$ </div> <div style="margin-left: 5px;">6</div> </div>	
	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; padding-left: 5px; margin-right: 5px;"> $(\exists y: \neg y = b) (Db \wedge Dy)$ </div> <div style="margin-left: 5px;">4</div> </div>	
	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; padding-left: 5px; margin-right: 5px;"> $\neg c = b$ $Db \wedge Dc$ </div> <div style="margin-left: 5px;">(10) 5</div> </div>	
5 Ext	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; padding-left: 5px; margin-right: 5px;"> Db </div> <div style="margin-left: 5px;">(8)</div> </div>	
5 Ext	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; padding-left: 5px; margin-right: 5px;"> Dc </div> <div style="margin-left: 5px;">(9)</div> </div>	
	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; padding-left: 5px; margin-right: 5px;"> $(\forall y: \neg y = a) \neg Dy$ </div> <div style="margin-left: 5px;">b:8, c:9</div> </div>	
8 SC	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; padding-left: 5px; margin-right: 5px;"> $b = a$ </div> <div style="margin-left: 5px;">a—b, c</div> </div>	
9 SC	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; padding-left: 5px; margin-right: 5px;"> $c = a$ </div> <div style="margin-left: 5px;">a—b—c, (10)</div> </div>	
	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; padding-left: 5px; margin-right: 5px;"> \perp </div> <div style="margin-left: 5px;">7</div> </div>	
10 Nc=	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; padding-left: 5px; margin-right: 5px;"> $(\exists y: \neg y = a) Dy$ </div> <div style="margin-left: 5px;">6</div> </div>	
7 RNcP	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; padding-left: 5px; margin-right: 5px;"> \perp </div> <div style="margin-left: 5px;">4</div> </div>	
6 CR	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; padding-left: 5px; margin-right: 5px;"> \perp </div> <div style="margin-left: 5px;">3</div> </div>	
4 PRCh	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; padding-left: 5px; margin-right: 5px;"> \perp </div> <div style="margin-left: 5px;">2</div> </div>	
3 PCh	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; padding-left: 5px; margin-right: 5px;"> \perp </div> <div style="margin-left: 5px;">1</div> </div>	
2 PCh	$\neg \exists x (\exists y: \neg y = x) (Dx \wedge Dy)$	
1 RAA		

At most one thing is done

At stage 10, the conclusion \perp could also be justified as coming by DC from $\neg c = b$ alone since $c = a$ serves to make b and c co-aliases.

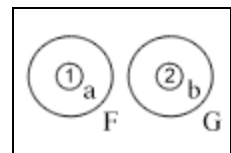
At most one thing is done

	$\neg \exists x (\exists y: \neg y = x) (Dx \wedge Dy)$	(9)
	$\forall x (\exists y: \neg y = x) Dy$	a:2, b:4
2 UI	$(\exists y: \neg y = a) Dy$	3
	<div style="display: flex; align-items: center;"> <div style="border: 1px solid black; border-radius: 50%; padding: 2px 5px; margin-right: 5px;">b</div> <div style="border-left: 1px solid black; padding-left: 5px;"> $\neg b = a$ Db </div> </div>	(6)
4 UI	$(\exists y: \neg y = b) Dy$	5
	<div style="display: flex; align-items: center;"> <div style="border: 1px solid black; border-radius: 50%; padding: 2px 5px; margin-right: 5px;">c</div> <div style="border-left: 1px solid black; padding-left: 5px;"> $\neg c = b$ Dc </div> </div>	(7) (6)
6 Adj	$Db \wedge Dc$	X, (7)
7 REG	$(\exists y: \neg y = b) (Db \wedge Dy)$	X, (8)
8 EG	$\exists x (\exists y: \neg y = x) (Dx \wedge Dy)$	X, (9)
	•	
9 Nc	\perp	5
5 PRCh	\perp	3
3 PRCh	\perp	1
1 NcP	$\exists x \neg (\exists y: \neg y = x) Dy$	

Something is such that nothing other than it is done

2.a.

	$\exists x Fx$	1
	$\exists x Gx$	2
	<div style="display: flex; align-items: center;"> <div style="border: 1px solid black; border-radius: 50%; padding: 2px 5px; margin-right: 5px;">a</div> <div style="border-left: 1px solid black; padding-left: 5px;">Fa</div> </div>	(5)
	<div style="display: flex; align-items: center;"> <div style="border: 1px solid black; border-radius: 50%; padding: 2px 5px; margin-right: 5px;">b</div> <div style="border-left: 1px solid black; padding-left: 5px;">Gb</div> </div>	(7)
	$\forall x \neg (Fx \wedge Gx)$	a:4, b:6
4 UI	$\neg (Fa \wedge Ga)$	5
5 MPT	$\neg Ga$	
6 UI	$\neg (Fb \wedge Gb)$	7
7 MPT	$\neg Fb$	
	○	Fa, $\neg Fb$, $\neg Ga$, Gb $\Rightarrow \perp$
	\perp	3
3 NcP	$\exists x (Fx \wedge Gx)$	2
2 PCh	$\exists x (Fx \wedge Gx)$	1
1 PCh	$\exists x (Fx \wedge Gx)$	



b.

	$(\exists x: Fx) Gx$	1
	$(\exists x: Fx) Hx$	2
	$(\forall x: Fx) (\forall y: Fy) x = y$	a:3
	⊖ a	
	Fa	
	Ga	(7)
	⊖ b	
	Fb	
	Hb	(8)
3 SB	$(\forall y: Fy) a = y$	b:4
4 SB	a = b	a—b
	$\forall x \neg (Gx \wedge Hx)$	a:6
6 UI	$\neg (Ga \wedge Ha)$	7
7 MPT	$\neg Ha$	(8)
	•	
	⊥	5
8 Nc=	$\exists x (Gx \wedge Hx)$	2
5 NcP	$\exists x (Gx \wedge Hx)$	1
2 PRCh	$\exists x (Gx \wedge Hx)$	1
1 PRCh	$\exists x (Gx \wedge Hx)$	

- 3. a.** *Some road sign was colored*
Every road sign was a traffic marker
If anything was colored, it was painted

	$(\exists x: Sx) Cx$	1
	$(\forall x: Sx) Tx$	a:2
	$\forall x (Cx \rightarrow Px)$	a:3
	⊖ a	
	Sa	(2)
	Ca	(4)
2 SB	Ta	(5)
3 UI	$Ca \rightarrow Pa$	4
4 MPP	Pa	(5)
5 REG	$(\exists x: Tx) Px$	X, (6)
	•	
6 QED	$(\exists x: Tx) Px$	1
1 PRCh	$(\exists x: Tx) Px$	

Some traffic marker was painted

- b.** *Someone who owns a snake was pleased*
Every snake is a reptile

	$(\exists x: Px \wedge (\exists y: Sy) Oxy) Dx$ $(\forall x: Sx) Rx$	1 b:4
	a	
	$Pa \wedge (\exists y: Sy) Oay$ Da	2 (7)
2 Ext	Pa	(6)
2 Ext	$(\exists y: Sy) Oay$	3
	b	
	Sb Oab	(4) (5)
4 SB	Rb	(5)
5 REG	$(\exists y: Ry) Oay$	X, (6)
6 Adj	$Pa \wedge (\exists y: Ry) Oay$	X, (7)
7 REG	$(\exists x: Px \wedge (\exists y: Ry) Oxy) Dx$	X, (8)
	•	
8 QED	$(\exists x: Px \wedge (\exists y: Ry) Oxy) Dx$	3
3 PRCh	$(\exists x: Px \wedge (\exists y: Ry) Oxy) Dx$	1
1 PRCh	$(\exists x: Px \wedge (\exists y: Ry) Oxy) Dx$	

Someone who owns a reptile was pleased