

8.5.x. Exercise questions

1. Use the system of derivations to establish each of the following:
 - a. $\exists x Fx, \forall x (Fx \rightarrow Gx) \Rightarrow \exists x Gx$
 - b. $(\exists x: Fx) Gx, (\forall x: Gx) Hx \Rightarrow (\exists x: Fx) Hx$
 - c. $\forall x (Fx \rightarrow Ga) \Leftrightarrow \exists x Fx \rightarrow Ga$
 - d. $Fa \Leftrightarrow (\exists x: x = a) Fx$
 - e. $(\exists x: Fx) \forall y Rxy \Rightarrow \forall x (\exists y: Fy) Ryx$
 - f. $(\exists x: Gx) Fx, \neg Fa \Rightarrow (\exists x: \neg x = a) Gx$
 - g. $\forall x (Fx \rightarrow Ga), \forall x (Ga \rightarrow Fx), \exists x Fx \Rightarrow \forall x Fx$
 - h. *Everyone loves everyone who loves anyone, Someone loves someone \Rightarrow Everyone loves everyone*
 - i. *Something is such that nothing other than it is done \Leftrightarrow At most one thing is done*

2. Use derivations to check each of the claims below; if a derivation indicates that a claim fails, describe a structure that divides an open gap. You need not worry about infinite derivations.
 - a. $\exists x Fx, \exists x Gx \Rightarrow \exists x (Fx \wedge Gx)$
 - b. $(\exists x: Fx) Gx, (\exists x: Fx) Hx, (\forall x: Fx) (\forall y: Fy) x = y \Rightarrow \exists x (Gx \wedge Hx)$

3. In the following, choose one of each bracketed pair of premises and one each bracketed pair of words or phrases in the conclusion so as to make a valid argument; then analyze the premises and conclusion and construct a derivation to show that the argument is valid.
 - a. *Some road sign was colored*
 $[$ *Every stop sign was a road sign* $|$ *Every road sign was a traffic marker* $]$
 $[$ *If anything was red, it was colored* $|$ *If anything was colored, it was painted* $]$

Some $[$ *stop sign* $|$ *traffic marker* $]$ *was* $[$ *red* $|$ *painted* $]$
 - b. *Someone who owns a snake was pleased*
 $[$ *Every cobra is a snake* $|$ *Every snake is a reptile* $]$

Someone who owns a $[$ *cobra* $|$ *reptile* $]$ *was pleased*