8.4. Definite descriptions

8.4.0. Overview

Up to this point, we have analyzed definite descriptions only by identifying component individual terms; now we will consider two ways of analyzing them to identify the descriptions from which they are formed.

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8.4.1. Definite descriptions as quantifier phrases

We have been treating definite descriptions as individual terms and analyzing them only by extracting component terms. In the early years of the 20th century the British logician and philosopher Bertrand Russell (1872-1970) proposed a way of analyzing definite descriptions that, in effect, treats them as quantifier phrases. For example, he would treat the sentence *The house Jack built still stands* as making a claim that could be stated more explicitly as:

Something such that it and only it is a house Jack built is such that (it still stands)

If we make this restatement the starting point of a symbolic analysis, we will get the following:

The house Jack built still stands
Something such that it and only it is a house Jack built is such that (it still stands)
(∃x: x *and only* x *is a house Jack built*) x *still stands*
(∃x: x *is a house Jack built* ∧ *only* x *is a house Jack built*) Sx
(∃x: (x *is a house* ∧ *Jack built* x) ∧ *only a thing identical to* x *is such that (it is a house Jack built)*) Sx
(∃x: (Hx ∧ Bjx) ∧ (∀y: ¬ y = x) ¬ y *is a house Jack built*) Sx
(∃x: (Hx ∧ Bjx) ∧ (∀y: ¬ y = x) ¬ (y *is a house* ∧ *Jack built* y)) Sx

(∃x: (Hx ∧ Bjx) ∧ (∀y: ¬ y = x) ¬ (Hy ∧ Bjy)) Sx
∃x ((Hx ∧ Bjx) ∧ ∀y (¬ y = x → ¬ (Hy ∧ Bjy)) ∧ Sx)

[B: λxy (x *built* y); H: λx (x *is house*); S: λx (x *still stands*); j: *Jack*]

Notice that the sentence *A house Jack built still stands* could be restated as *Something such that it is house Jack built is such that (it still stands)*, so the difference between the indefinite and definite article on Russell's analysis lies in the extra phrase *and only it*. In the analysis above, that phrase yields an added conjunct in the restricting formula that appears in English as *only* x *is a house Jack built* and in symbols as (∀y: ¬ y = x) ¬ (Hy ∧ Bjy). This reflects the requirement of uniqueness noted in 6.2.1 as a condition for the reference of definite descriptions, and the analysis above entails *Jack built at most one house*.

Notice that Russell does not treat *The house Jack built still stands* as *Exactly one house Jack built still stands*. The latter sentence makes a claim of uniqueness, too, but a weaker one. It entails only *Jack built at most one house that still stands* and not *Jack built at most one house*. Russell's analysis also entails *Any house Jack built still stands* and this means that, with a little artificiality, the difference between it and the weaker claim of uniqueness can be expressed as the difference between a non-restrictive and a restrictive relative clause—i.e., between the stronger *The houses Jack built, which still stand, number one* and the weaker *The houses Jack built that still stand number one*. Notice that the first of these cannot be treated as a simple claim that there is exactly one example of a certain sort.

In general, Russell recommended that we analyze a sentence of the form *The* C *is such that (… it …)* as equivalent to

(∃x: x *is a* C ∧ (∀y: ¬ y = x) ¬ y *is a* C) … x …

—i.e., as we might analyze *Something such that it and only it is a* C *is such that (… it…)*. It is sometimes convenient to use instead the shorter form

(∃x: x *is a* C ∧ (∀y: y *is a* C) x = y) … x …

which amounts to *Some* C *that is all the* C*s there are is such that (… it…)*. As was noted in 8.3.3 for a similar restatement of sentences using *exactly 1*, this is equivalent to the first form by principle of contraposition and the symmetry of identity.

Russell's analysis of definite descriptions has been widely accepted, but it is not uncontroversial since it opens up the possibility of scope ambiguities that many do not find in sentences involving definite descriptions. In particular, if we analyze a negative sentence containing a definite description using Russell's approach, we can regard the negation either as the main logical operator or as a part of the quantified predicate that is left when we remove the definite description. To choose one of Russell's own examples, we could regard *The present king of France is not bald* as making either of the claims below.

¬ *the present king of France is bald*
The present king of France is such that he is not bald

Russell's analysis of the positive claim *The present king of France is bald* implies that there is at present a king of France, so it is false (and was false already when Russell proposed the analysis). Russell then held that the first of the sentences above is true because it is the negation of a false statement. But, by the same token, the second sentence claims in part that there is presently a king of France, so it is false on his view. Thus *The present king of France is not bald* is, on Russell's analysis, open to two interpretations, one on which it is true and another of which it is false, and many philosophers have found no such ambiguity in the sentence. Indeed, many would claim that the sentence is neither true nor false since the definite description *the present king of France* does not refer to anything.

8.4.2. Definite descriptions as individual terms

Prior to 8.4.1, we had treated definite descriptions as individual terms, understanding them to have at least the nil value as a reference value. Historically, this approach is associated with Frege, who suggested that an actual object—for example, the number 0—be stipulated as the reference of definite descriptions that did not otherwise have one.

It is possible to retain the view that definite descriptions are individual terms and still go on to analyze them in a way that exposes the component descriptions; but, to do this, we need to introduce some further notation. This is a logical operation, a ***description operator***, that applies to a predicate abstract to form an individual term. Our notation will be a sans-serif capital ℩ and we will abbreviate ℩[λx ρx] as ℩x ρx. This notation might be read in English as *the thing* x *such that* ρx. Notice that this is a noun phrase rather than a sentence, so, although the description operator looks like an unrestricted quantifier, its reading does not involve a verb.

The reference value of ℩x ρx is stipulated to be the one value in the extension of ρ if contains just one value and to be the nil value otherwise. We do not distinguish the nil value from others in a referential range in any other way, so the stipulation of it as the default value of ℩x ρx is somewhat limited in its significance. But this stipulation does entail that definite descriptions that fail to uniquely describe an object all have the same reference value. The description *the rational number whose square is 2* thus has the same reference value as *the planet whose orbit lies between the Earth and Venus.*

If we use the description operator to analyze *The house Jack built still stands*, we get

The house Jack built still stands
S *the house Jack built*
S(℩x (x *is a house Jack built*))
S(℩x (x *is a house* ∧ *Jack built* x))

S(℩x (Hx ∧ Bjx))

[B: λxy (x *built* y); H: λx (x *is house*); S: λx (x *still stands*); j: *Jack*]

The parentheses surrounding the whole definite description in this analysis are not needed to avoid ambiguity in our notation, but they make it easier to read.

This analysis does more than use different notation from Russell's analysis; it offers a different interpretation of the sentence. While the simpler notation may be pleasing, the interpretation may not be, so we should consider it more closely. To compare the two interpretations, it will help to give Russell's in a different but equivalent form. Since on Russell's analysis *The* C *is such that (… it …)* entails both *Some* C *is such that (… it …)* and that at most one thing is a C, it can be restated somewhat redundantly as the conjunction

There is exactly one C ∧ *some* C *is such that (… it …)*

That is, Russell interprets *The house Jack built still stands* as *There is exactly one house that Jack built and some house that Jack built still stands*.

On the other hand, if we analyze *The* C *is such that (… it …)* using the description operator, we interpret it as saying that the predicate λx (… x …) is true of the reference value of *the* C. Now, what that reference value is depends on whether *There is exactly one* C is true. If there is exactly one C, the value of *the* C is the one and only C. Otherwise, the value of *the* C is the nil value. For example, if Jack did build exactly one house, the sentence *The house Jack built still stands* is true just in case this house still stands. But if Jack built no house or more than one, this sentence is true if and only if the predicate λx (x *still stands*) is true of the nil value.

To make it easier to express this interpretation in English, let's fix an individual term whose reference is bound to be nil and read it in English as *the nil*. Since the extension of λx ⊥ is bound to be empty, the definite description ℩x ⊥ could play this role, but it will be convenient to have a special symbol, for which we will use ∗ (known as the ***asterisk operator***).

Then we can express the content of the analysis using the description operator as follows:

(*there is exactly one* C ∧ *some* C *is such that* (… *it* …))
∨ (¬ *there is exactly one* C ∧ … ∗ …)

Comparison with the expression of Russell's analysis given above will show that this interpretation is weaker, having been hedged by an added disjunct. It could be expressed equivalently as follows:

If there is exactly one C, *then some* C *is such that* (… *it* …);
otherwise, … ∗ …

where the English *if* ϕ *then* ψ*; otherwise* χ expresses the form

$(\phi \rightarrow \psi) \wedge (\neg \phi \rightarrow \chi)$, which we have called a branching conditional. This is equivalent to the form $(\phi \wedge \psi) \vee (\neg \phi \wedge \chi)$ that was used above because each form has the same truth value as ψ when ϕ is true and the same value as χ when ϕ is false. While, the formulation of the content of this analysis using the branching conditional makes the comparison with Russell's analysis a little less direct, it is probably the more natural way of thinking about the significance of this approach to definite descriptions in its own right.

So, when we use the description operator, we interpret *The house Jack built still stands* as either of the following equivalent claims:

Either there is exactly one house that Jack built and some house that Jack built still stands; or there is not exactly one and the nil still stands

If there is exactly one house that Jack built then some house that Jack built still stands; otherwise the nil still stands

This interpretation has both fortunate and unfortunate consequences.

First the bad news. Because the analysis using the description operator hedges the claim it makes with the possibility that there is not exactly one house that Jack built, it can be true if he built no house or more than one. So we must ask whether we would count the original sentence as true in this sort of case. In answering this question, it is important to remember that the analysis will be true in such a case only if the predicate λx (x *still stands*) is true of the nil reference value. The truth value yielded by properties when they are applied to the nil value is something that we have left open. (More precisely, this is true in the case of unanalyzed predicates; λx x = x, for example, is bound to be true of the nil value because it is true of all reference values.) So when we analyze definite descriptions using the description operator, we do not specify the truth value of *The house Jack built still stands* in cases where *the house Jack built* does not refer. But on Russell's account the value is definitely **F** in these cases. If the discussion of the issue throughout the course of the last century has shown anything, it has shown that there is no consensus on this matter among the community of English speakers.

That's the bad news. The good news is that the analysis using the description operator removes any room for ambiguity concerning the relative scope of definite descriptions and negation. That much is clear just from the notation. The definite description operator forms terms and to deny that a predicate applies to a term is the same thing as to apply a negative predicate. That is, $\neg\, \theta\tau \Leftrightarrow [\lambda x\ \neg\, \theta x]\tau$. (Indeed, we really have more than an equivalence here since we regard these symbolic forms as notation for the same sentence.)

We can see this lack of ambiguity also by exploring the interpretation given by the second analysis. First, let us look a little more closely at the ambiguity exhibited by *The present king of France is not bald* on Russell's analysis. Consider the following restatements and partial analyses of a pair of sentences:

The present king of France is such that (he is bald)
There is at present one and only one king of France
∧ *some present king of France is such that (he is bald)*

$$O \wedge (\exists x{:}\ Kx)\ Bx$$

The present king of France is such that (he is not bald)
There is at present one and only one king of France
∧ *some present king of France is such that (he is not bald)*

$$O \wedge (\exists x{:}\ Kx)\ \neg\, Bx$$

[B: λx (x *is bald*); K: λx (x *is at present king of France*);
O: *there is at present one and only one king of France*]

If O is true, at least one of these is true because there is some king of France at present who must be either bald or not, and at most one is true because there is no more than one present king of France so being bald and not being bald cannot both be exemplified by present kings of France. But, if O is not true, both of the sentences above are false; and therefore they are not contradictory. Now, on Russell's analysis, *The present king of France is not bald* might be interpreted as equivalent to either $\neg\, (O \wedge (\exists x{:}\ Kx)\ Bx)$, the denial of the first sentence above, or $O \wedge (\exists x{:}\ Kx)\ \neg\, Bx$, the second sentence. And these two interpretations are not equivalent because the two sentences above are not contradictory.

On the other hand if we consider the same two sentences but restate them in the way corresponding to the semantics of the definite description operator we get this:

The present king of France is such that (he is bald)
(O ∧ *some present king of France is such that (he is bald)*)
∨ (¬ O ∧ *the nil is bald*)

$$(O \wedge (\exists x{:}\ Kx)\ Bx) \vee (\neg\, O \wedge B*)$$

The present king of France is such that (he is not bald)
(O ∧ *some present king of France is such that (he is not bald)*)
∨ (¬ O ∧ *the nil is not bald*)

$$(O \wedge (\exists x{:}\ Kx)\ \neg\, Bx) \vee (\neg\, O \wedge \neg\, B*)$$

Now, we have already seen that, if O is true, the left disjunct of exactly one of these is true and, since the right disjuncts are both false when O is true, exactly one of the disjunctions will be true in such a case. And, when O is false, the left disjuncts are both false and exactly one of the right disjuncts is true. So again exactly one the disjunctions is true, and these sentences are contradictory. Thus, the denial of the first of these sentences is equivalent to the second; and taking *The present king of France is not bald* to be a negation leads to the same interpretation as we would get by supposing that it applies the negative predicate λx (x *is not bald*) to the individual term *the present king of France*.

In an analysis using the description operator, both of the sentences we have been considering are given weaker interpretations than Russell would give them, and these interpretations are weaker in different ways. In particular, in a case where O is false, one of the hedges is true and the other is not. Which is which depends on whether λx (x *is bald*) is true or false of the nil value, but we do not care which hedge is true and which false. What is important is that, when the sentence O is false and thus both of the logical forms derived from Russell's analysis are false, one and only one of the weaker pair of forms is true.

8.4.3. Examples: restrictive *vs.* non-restrictive relative clauses

The distinction between restrictive and non-restrictive relative clauses is a natural application of an analysis of definite descriptions. Although the significance of the distinction is not as great as it is for generalizations, it is greater for definite descriptions than it is for claims of exemplification. And, since restrictive relative clauses are part of definite descriptions but not themselves individual terms, it is only the sort of analysis of definite descriptions that we are now considering that can exhibit their role.

We will consider a single pair of sentences and analyze each of them using the two approaches to definite descriptions. Since these analyses are not equivalent, we can expect different results but, since the difference between the analyses involves a failure of normal reference, we cannot expect great differences when the descriptions work normally—i.e., when reference succeeds and true claims are made.

The two sentences we will consider are these:

The part that Tom requested was defective.
The part, which Tom requested, was defective.

The difference between having a restrictive relative clause in the first and a non-restrictive relative clause in the second is, intuitively, whether the relative clause contributes to the specification of what is referred to or instead to what is said about it. That difference can be emphasized by expanding the second sentence to *The part, which, by the way, Tom requested, was defective*.

We will begin with an analysis of these two sentences using the description operator. This begins as an analysis in chapter 6 would have but continues further. In the case of the first sentence, we have

The part that Tom requested was defective
The part that Tom requested *was defective*
D *the part that Tom requested*
D(Ix x *is a part that Tom requested*)
D(Ix (x *is a part* ∧ *Tom requested* x))

D(Ix (Px ∧ Rtx))

[D: λx (x *was defective*); P: λx (x *is a part*); R: λxy (x *requested* y); t: *Tom*]

In chapter 6, we would have ended up with something like D(pt) where p abbreviated a functor that produced the term *the part that Tom requested* when applied to the term *Tom*. Since the two expressions

[λy (Ix (Px ∧ Ryx))]t   Ix (Px ∧ Rtx)

are really two forms of notation for the same term, we can say that the analysis above extends the analysis of chapter 6 by analyzing the functor p as λy (℩x (Px ∧ Ryx)). Indeed, one of the main reasons definite descriptions were of interest to Frege and Russell was their role in specifying the output of functors by way of relations since many mathematical functions are naturally defined in this way.

The analysis of the sentence with non-restrictive relative clause also begins as in chapter 6.

The part, which Tom requested, was defective
Tom *requested* *the part* ∧ *the part* *was defective*
R *Tom* *the part* ∧ D *the part*
Rt(℩x x *is a part*) ∧ D(℩x x *is a part*)

$$Rt(\iota x\, Px) \wedge D(\iota x\, Px)$$

[D: λx (x *was defective*); P: λx (x *is a part*); R: λxy (x *requested* y); t: *Tom*]

To make it easier to compare the two analyses, let us reorder the conjuncts in the second to get

$$D(\iota x\, Px) \wedge Rt(\iota x\, Px)$$

and then restate this using an abstract so that the definite description occurs only once

$$[\lambda x\, (Dx \wedge Rtx)](\iota x\, Px)$$

The difference between the sentences restrictive and non-restrictive clauses, when seen in this way—i.e., as

$$D(\iota x\, (Px \wedge Rtx)) \qquad [\lambda x\, (Dx \wedge Rtx)](\iota x\, Px)$$

—lies in the location of the predicate λx Rtx or λx (*Tom requested* x). In both cases it is used to provide a further conjunct; but, in the analysis of restrictive clause, this conjunct appears in the description to which the definite article is applied, and in the analysis of the non-restrictive clause, it appears in what is predicated of a definite description. This is the symbolic analogue of the idea that restrictive relative clause contributes to determining the reference of an individual term while a non-restrictive clause adds to what is said about the term's referent.

We can expect to find something similar when apply Russell's analysis. In the case of the first sentence, we get

The part that Tom requested was defective
The part that Tom requested is such that (it was defective)
(∃x: x *is a part that Tom requested* ∧ (∀y: ¬ y = x) ¬ y *is a part the Tom requested*) x *was defective*
(∃x: (x *is a part* ∧ *Tom requested* x) ∧ (∀y: ¬ y = x) ¬ (y *is a part* ∧ *Tom requested* y)) x was defective

$$(\exists x{:}\ (Px \wedge Rtx) \wedge (\forall y{:}\ \neg\, y = x)\ \neg\, (Py \wedge Rty))\ Dx$$
or: $(\exists x{:}\ (Px \wedge Rtx) \wedge (\forall y{:}\ Py \wedge Rty)\ x = y)\ Dx$

[D: λx (x *was defective*); P: λx (x *is a part*); R: λxy (x *requested* y); t: *Tom*]

On Russell's analysis, the definite article *the* can be seen to mark an operation that applies to a predicate ρ to form the quantifier (∃x: ρx ∧ (∀y: ¬ y = x) ¬ ρy) or (∃x: ρx ∧ (∀y: ρy) x = y). In the sentence with the restrictive relative clause, the predicate ρ is λx (Px ∧ Rtx), and this involves the predicate λx Rtx that corresponds to the relative clause.

Russell's analysis of the sentence with a non-restrictive relative clause finds a conjunction. We must choose whether this conjunction has wider or narrower scope than the quantifier phrase associated with the definite description; but, while this is the sort of thing that leads to non-equivalent analyses in the case of negation, here the results of the two approaches are equivalent.

The part, which Tom requested, was defective
Tom requested the part ∧ *the part was defective*
the part is such that (Tom requested it) ∧ *the part is such that (it was defective)*
(∃x: x *is a part* ∧ (∀y: ¬ y = x) ¬ y *is a part*) *Tom requested* x ∧ (∃x: x *is a part* ∧ (∀y: ¬ y = x) ¬ y *is a part*) x *was defective*

$$(\exists x{:}\ Px \wedge (\forall y{:}\ \neg\, y = x)\ \neg\, Py)\ Rtx \wedge (\exists x{:}\ Px \wedge (\forall y{:}\ \neg\, y = x)\ \neg\, Py)\ Dx$$
or: $(\exists x{:}\ Px \wedge (\forall y{:}\ Py)\ x = y)\ Rtx \wedge (\exists x{:}\ Px \wedge (\forall y{:}\ Py)\ x = y)\ Dx$

[D: λx (x *was defective*); P: λx (x *is a part*); R: λxy (x *requested* y); t: *Tom*]

The part, which Tom requested, was defective
The part is such that (it, which Tom requested, was defective)
(∃x: x *is a part* ∧ (∀y: ¬ y = x) ¬ y *is a part*) x*, which Tom requested, was defective*
(∃x: Px ∧ (∀y: ¬ y = x) ¬ Py) (*Tom requested* x ∧ x *was defective*)

$$(\exists x{:}\ Px \wedge (\forall y{:}\ \neg\, y = x)\ \neg\, Py)\ (Rtx \wedge Dx)$$
or: $(\exists x{:}\ Px \wedge (\forall y{:}\ Py)\ x = y)\ (Rtx \wedge Dx)$

It usually makes a difference whether an existential is applied to a conjunction or to each conjunct separately because different examples may make each conjunct true and there may be no one example that would serve for both. But this will not happen with the sort of existential quantifiers used to represent definite descriptions because the restricting formula requires uniqueness. This means that if we claim the existence of an example x that satisfies this restricting formula, part of what we have claimed is that this is the only example possible. So, when the quantifier is applied to separately in two conjuncts of a conjunctions, the conjuncts cannot be true unless there is a single example which makes both true.

Here is a table showing the simplest analyses of each of the four sorts:

	restrictive clause	*non-restrictive clause*
description operator	D(℩x (Px ∧ Rtx))	Rt(℩x Px) ∧ D(℩x Px)
Russell's analysis	(∃x: (Px ∧ Rtx) ∧ (∀y: Py ∧ Rty) x = y) Dx	(∃x: Px ∧ (∀y: Py) x = y) (Rtx ∧ Dx)

The differences between the two sorts of relative clause are easiest to describe in the case of Russell's analysis. If we convert the analyses in the second row to unrestricted existential quantifiers and reorder conjuncts, we get the following:

restrictive clause	∃x (Px ∧ Rtx ∧ Dx ∧ (∀y: Py ∧ Rty) x = y)
non-restrictive clause	∃x (Px ∧ Rtx ∧ Dx ∧ (∀y: Py) x = y)

This reformulation makes it clear that the sentence stated using the restrictive relative clause is entailed by the sentence using the non-restrictive clause. The only difference lies in the restriction of the generalization appearing as the last conjunct of the formula to which the existential is applied. And the added restriction makes the generalization derived from the restrictive clause weaker since it says about x only that it accounts for all the parts Tom requested rather than that it accounts for all the parts whatsoever. And it is also clear for the same reason that no entailment holds in the other direction.

In the analyses using the description operator, if a description is not uniquely satisfied, the definite description has the nil reference value and whether what is said is true or false will depend on what predicates are true or false of the nil value. Each of the two sorts of clause succeeds in referring in some circumstances where the other does not: there may be more than one part but just one that Tom requested and there may be exactly one part but none that Tom requested. It follows that there will be some circumstance in which the two sentences will be talking about different things, in one case a real object and, in the other, the nil value. It is easy to find such circumstances in which the sentence with the restrictive clause is true and the one with the non-restrictive clause is false. The other direction is harder; but, if there is more than one part and Tom requested only one, which was not defective, D(℩x (Px ∧ Rtx)) will be false and Rt(℩x Px) ∧ D(℩x Px) will still be true provided that Rt∗ and D∗ are true. (Notice that it must then be the case that P∗ is false if ℩x (Px ∧ Rtx) is to have a non-nil value.) Since it is also easy to find cases where the two sentences are both true or both false on when they are analyzed using the description operator, the two sentences are counted as logically independent by that analysis.

Thus, while each sort of analysis makes a distinction between the meanings of the two sentences, their accounts of this distinction are different. However, this difference does not provide much basis on which to argue for the correctness of one analysis over the other. The difference only concerns circumstances in which a definite description is not uniquely satisfied; and, although we are assuming that a sentence containing such a description does have a truth value, there seems to be little grounds for saying what that truth value ought to be.

The most important differences between restrictive and non-restrictive clauses are probably not the differences in truth conditions that our symbolic analyses are designed to capture but instead differences in appropriateness. To be used appropriately, a definite description must do enough to identify a unique object given the context, and there seem to be also requirements governing the way this is done.

First, the description should identify an object using information that is already shared by parties to the conversation. It would be odd to use *The part that Tom requested was defective* if one's audience was not already aware that Tom had requested a part—it might prompt the response, *I didn't know Tom requested a part*—but *The part, which Tom requested, was defective* would be appropriate in this sort of case if the part in question was already sufficiently salient that it could be identified by the simple description *the part*. On the other hand, if it is known that Tom requested one and only one part but this part is not already sufficiently salient to be identified as *the part*, the sentence with the restrictive clause is appropriate but the one with the non-restrictive clause would not be. In a case like this, the added description provided by the restrictive clause might serve to distinguish one part among a group of equally salient parts or to shift attention from one part to another one.

It also seems to be a requirement for appropriateness that the various elements of a definite description actually be needed to identify an object. If a single part is salient enough to be identified by *the part* alone, then, even if everyone is aware that Tom had requested it, the sentence *The part that Tom requested was defective* will seem odd and may prompt the response *But I thought that was the part we were talking about*. The sentence with the non-restrictive clause would be no better under these circumstances but for a different reason: it would purport to introduce as new information something that was already known.

8.4.s. Summary

A famous analysis of definite descriptions was first proposed early in the 20th century by Bertrand Russell. According to Russell's analysis, a sentence *The* C *is such that (… it …)* amounts to *Something such that it and only it is a* C *is such that (… it …)*. This analysis is equivalent to the conjunction of *Some* C *is such that (… it …)* and *There is at most one* C, so, according to Russell, the effect of using a definite rather than an indefinite article is to imply the latter conjunct. Russell's analysis treats a definite description as a kind of quantifier phrase and leads to scope ambiguities in negative sentences involving definite descriptions.

An alternative approach avoids this suggestion of ambiguity by treating definite descriptions as individual terms and analyzing them by the use of a description operator, which applies to predicate abstracts to form terms. We use a sans-serif capital I as notation for the description operator, abbreviating I[λx ρx] by Ix ρx. A term formed in this way has the sole member of the predicate's extension as its reference value if that extension has a unique member; otherwise, its reference value is the nil value. We fix a logically constant term, the nil, which always has the nil value and use the notation ∗ (asterisk operator) for it. The content of *… the* C *…* on this analysis can be expressed using a branching conditional as *if there is exactly one* C*, then some* C *is such that (… it …); otherwise, … the nil …*.

Each of the two approaches to analyzing definite descriptions can be used to exhibit the difference between a restrictive and a non-restrictive relative clause when these modify a common noun governed by the article *the*. Although both analyses point to differences between such sentences, their accounts of the relations between them differ.

8.4.x. Exercise questions

1. Analyze the following in as much detail as possible; analyze definite descriptions in two ways, using Russell's approach and using the description operator.

a. *Sam guessed the winning number*.

b. *The winner who spoke to Tom was well-known*.

c. *The winner, who spoke to Tom, was well-known*.

d. *Every number greater than one is greater than its (own) positive square root*.

2. Synthesize idiomatic English sentences that express the propositions associated with the logical forms below using the intensional interpretations that follow them. You may use definite descriptions to express the sort of logical forms Russell's analysis produces.

a. (∃x: Oxs ∧ (∀y: ¬ y = x) ¬ Oys) Cx
[C: λx (x *called*); O: λxy (x *owns* y); s: *Spot*]

b. Fj(Ix (Hx ∧ Ex(Iy Pyj)))
[E: λxy (x *enlarged* y); F: λxy (x *found* y); H: λx (x *is a photographer*); P: λxy (x *is a picture of* y); j: *John*]

8.4.xa. Exercise answers

1. **a.** *using Russell's analysis:*

Sam guessed the winning number
the winning number is such that (Sam guessed it)
(∃x: x *is a winning number* ∧ *only* x *is a winning number*) *Sam guessed* x
(∃x: Wx ∧ (∀y: ¬ y = x) ¬ y *is a winning number*) Gsx

$$(\exists x\colon Wx \land (\forall y\colon \neg\, y = x)\ \neg\, Wy)\ Gsx$$
$$\exists x\, (Wx \land \forall y\, (\neg\, y = x \rightarrow \neg\, Wy) \land Gsx)$$
or:
$$(\exists x\colon Wx \land (\forall y\colon Wy)\ x = y)\ Gsx$$
$$\exists x\, (Wx \land \forall y\, (Wy \rightarrow x = y) \land Gsx)$$

[G: λxy (x *guessed* y); W: λx (x *is a winning number*); s: *Sam*]
[*Note:* λx (x *is a winning number*) might be open to further analysis as λx (x *is a number* ∧ x *won*)]

with the description operator:
Sam guessed the winning number
G *Sam* *the winning number*
Gs(℩x x *is a winning number*)

$$Gs(\iota x\ Wx)$$

b. *using Russell's analysis:*

The winner who spoke to Tom was well-known
The winner who spoke to Tom is such that (he or she was well-known)
(∃x: x *is a winner who spoke to Tom* ∧ *only* x *is a winner who spoke to Tom*) x *was well-known*
(∃x: (x *is a winner* ∧ x *spoke to Tom*) ∧ (∀y: ¬ y = x) ¬ (y *is a winner* ∧ y *spoke to Tom*)) Kx

$$(\exists x\colon (Wx \land Sxt) \land (\forall y\colon \neg\, y = x)\ \neg\, (Wy \land Syt))\ Kx$$
$$\exists x\, ((Wx \land Sxt) \land \forall y\, (\neg\, y = x \rightarrow \neg\, (Wy \land Syt)) \land Kx)$$
or:
$$(\exists x\colon (Wx \land Sxt) \land (\forall y\colon Wy \land Syt)\ x = y)\ Kx$$
$$\exists x\, ((Wx \land Sxt) \land \forall y\, ((Wy \land Syt) \rightarrow x = y) \land Kx)$$

K: λx (x *was well-known*); S: λxy (x spoke to y); W: λx (x *is a winner*); t: *Tom*]

with the description operator:
The winner who spoke to Tom was well-known
The winner who spoke to Tom was well-known
K *the winner who spoke to Tom*
K(℩x (x *is a winner who spoke to Tom*))
K(℩x (x *is a winner* ∧ x *spoke to Tom*))

$$K(\iota x\ (Wx \land Sxt))$$

c. *using Russell's analysis:*

The winner, who spoke to Tom, was well-known.
The winner is such that (he or she, who spoke to Tom, was well-known).
(∃x: x *is a winner* ∧ *only* x *is a winner*) x, *who spoke to Tom, was well-known*)
(∃x: x *is a winner* ∧ (∀y: ¬ y = x) ¬ y *is a winner*) (x *spoke to Tom* ∧ x *was well-known*)

$$(\exists x\colon Wx \land (\forall y\colon \neg\, y = x)\ \neg\, Wy)\ (Sxt \land Kx)$$
$$\exists x\, (Wx \land \forall y\, (\neg\, y = x \rightarrow \neg\, Wy) \land (Sxt \land Kx))$$
or:
$$(\exists x\colon Wx \land (\forall y\colon Wy)\ x = y)\ (Sxt \land Kx)$$
$$\exists x\, (Wx \land \forall y\, (Wy \rightarrow x = y) \land (Sxt \land Kx))$$

[K: λx (x *was well-known*); S: λxy (x *spoke to* y); W: λx (x *is a winner*); t: *Tom*]

with the description operator:
The winner, who spoke to Tom, was well-known.
The winner spoke to Tom ∧ *the winner was well-known*
S *the winner* Tom ∧ K *the winner*
S(℩x x *is a winner*)t ∧ K(℩x x *is a winner*)

$$S(\iota x\ Wx)t \land K(\iota x\ Wx)$$

d. *using Russell's analysis:*

Every number greater than one is greater than its positive square root
(∀x: x *is a number greater than one*) x *is greater than its positive square root*
(∀x: x *is a number* ∧ x *is greater than one*) x *is greater than the positive square root of* x
(∀x: Nx ∧ Gxo) *the positive square root of* x *is such that (*x *is greater than it)*
(∀x: Nx ∧ Gxo) (∃y: y *is a positive square root of* x ∧ *only* y *is a positive square root of* x) x *is greater than* y

(∀x: Nx ∧ Gxo) (∃y: (y *is positive* ∧ y *is a square root of* x) ∧ (∀z: ¬ z = y) ¬ (z *is positive* ∧ z *is a square root of* x)) Gxy

(∀x: Nx ∧ Gxo) (∃y: (Py ∧ Syx) ∧ (∀z: ¬ z = y) ¬ (Pz ∧ Szx)) Gxy
∀x ((Nx ∧ Gxo) → ∃y ((Py ∧ Syx) ∧ ∀z (¬ z = y → ¬ (Pz ∧ Szx)) ∧ Gxy))

or:

(∀x: Nx ∧ Gxo) (∃y: (Py ∧ Syx) ∧ (∀z: Pz ∧ Szx) y = z) Gxy
∀x ((Nx ∧ Gxo) → ∃y ((Py ∧ Syx) ∧ ∀z ((Pz ∧ Szx) → y = z) ∧ Gxy))

[G: λx (x *is greater than* y); N: λx (x *is a number*); P: λx (x *is positive*); S: λxy (x *is a square root of* y)]

with the description operator:

Every number greater than one is greater than its positive square root
(∀x: x *is a number* ∧ x *is greater than one*) x *is greater than the positive square root of* x
(∀x: Nx ∧ Gxo) G x *the positive square root of* x
(∀x: Nx ∧ Gxo) Gx(ιy y *is a positive square root of* x)
(∀x: Nx ∧ Gxo) Gx(ιy (y *is a positive* ∧ y *is a square root of* x))

(∀x: Nx ∧ Gxo) Gx(ιy (Py ∧ Syx))
∀x ((Nx ∧ Gxo) → Gx(ιy (Py ∧ Syx)))

2. **a.** (∃x: x *owns Spot* ∧ (∀y: ¬ y = x) ¬ y *owns Spot*) x *called*
(∃x: x *owns Spot* ∧ *only* x *owns Spot*) x *called*
The owner of Spot is such that (it called)

Spot's owner called

b. *John found* (ιx (x *is a photographer* ∧ x *enlarged* (ιy y *is a picture of John*)))
John found (ιx (x *is a photographer* ∧ x *enlarged the picture of John*))
John found (ιx (x *is a photographer who enlarged the picture of John*))

John found the photographer who enlarged the picture of him

Glen Helman 26 Nov 2004