8.4.xa. Exercise answers

1. **a.** *using Russell's analysis:*

Sam guessed the winning number
the winning number is such that (Sam guessed it)
(∃x: x *is a winning number* ∧ *only* x *is a winning number*) *Sam guessed* x
(∃x: Wx ∧ (∀y: ¬ y = x) ¬ y *is a winning number*) Gsx

$$(\exists x\!: Wx \land (\forall y\!: \neg\, y = x)\ \neg Wy)\ Gsx$$
$$\exists x\, (Wx \land \forall y\, (\neg\, y = x \rightarrow \neg Wy) \land Gsx)$$
or:
$$(\exists x\!: Wx \land (\forall y\!: Wy)\ x = y)\ Gsx$$
$$\exists x\, (Wx \land \forall y\, (Wy \rightarrow x = y) \land Gsx)$$

[G: λxy (x *guessed* y); W: λx (x *is a winning number*); s: *Sam*]
[*Note:* λx (x *is a winning number*) might be open to further analysis as λx (x *is a number* ∧ x *won*)]

with the description operator:
Sam guessed the winning number
G *Sam the winning number*
Gs(℩x x *is a winning number*)

$$Gs(\iota x\, Wx)$$

b. *using Russell's analysis:*

The winner who spoke to Tom was well-known
The winner who spoke to Tom is such that (he or she was well-known)
(∃x: x *is a winner who spoke to Tom* ∧ *only* x *is a winner who spoke to Tom*) x *was well-known*
(∃x: (x *is a winner* ∧ x *spoke to Tom*) ∧ (∀y: ¬ y = x) ¬ (y *is a winner* ∧ y *spoke to Tom*)) Kx

$$(\exists x\!: (Wx \land Sxt) \land (\forall y\!: \neg\, y = x)\ \neg (Wy \land Syt))\ Kx$$
$$\exists x\, ((Wx \land Sxt) \land \forall y\, (\neg\, y = x \rightarrow \neg (Wy \land Syt)) \land Kx)$$
or:
$$(\exists x\!: (Wx \land Sxt) \land (\forall y\!: Wy \land Syt)\ x = y)\ Kx$$
$$\exists x\, ((Wx \land Sxt) \land \forall y\, ((Wy \land Syt) \rightarrow x = y) \land Kx)$$

K: λx (x *was well-known*); S: λxy (x spoke to y); W: λx (x *is a winner*); t: *Tom*] *with the description operator:*

The winner who spoke to Tom was well-known

The winner who spoke to Tom was well-known

K *the winner who spoke to Tom*

K(℩x (x *is a winner who spoke to Tom*))

K(℩x (x *is a winner* ∧ x *spoke to Tom*))

$$K(\iota x\ (Wx \land Sxt))$$

c. *using Russell's analysis:*

The winner, who spoke to Tom, was well-known.

The winner is such that (he or she, who spoke to Tom, was well-known).

(∃x: x *is a winner* ∧ *only* x *is a winner*) x, *who spoke to Tom, was well-known*)

(∃x: x *is a winner* ∧ (∀y: ¬ y = x) ¬ y *is a winner*) (x *spoke to Tom* ∧ x *was well-known*)

$$(\exists x{:}\ Wx \land (\forall y{:}\ \neg\, y = x)\ \neg\, Wy)\ (Sxt \land Kx)$$

$$\exists x\ (Wx \land \forall y\ (\neg\, y = x \rightarrow \neg\, Wy) \land (Sxt \land Kx))$$

or:

$$(\exists x{:}\ Wx \land (\forall y{:}\ Wy)\ x = y)\ (Sxt \land Kx)$$

$$\exists x\ (Wx \land \forall y\ (Wy \rightarrow x = y) \land (Sxt \land Kx))$$

[K: λx (x *was well-known*); S: λxy (x *spoke to* y); W: λx (x *is a winner*); t: *Tom*]

with the description operator:

The winner, who spoke to Tom, was well-known.

The winner spoke to Tom ∧ *the winner was well-known*

S *the winner* Tom ∧ K *the winner*

S(℩x x *is a winner*)t ∧ K(℩x x *is a winner*)

$$S(\iota x\ Wx)t \land K(\iota x\ Wx)$$

d. *using Russell's analysis:*

Every number greater than one is greater than its positive square root

(∀x: x *is a number greater than one*) x *is greater than its positive square root*

(∀x: x *is a number* ∧ x *is greater than one*) x *is greater than the positive square root of* x

(∀x: Nx ∧ Gxo) *the positive square root of* x *is such that (*x *is greater than it)*

(∀x: Nx ∧ Gxo) (∃y: y *is a positive square root of* x ∧ *only* y *is a positive square root of* x) x *is greater than* y

(∀x: Nx ∧ Gxo) (∃y: (y *is positive* ∧ y *is a square root of* x) ∧
(∀z: ¬ z = y) ¬ (z *is positive* ∧ z *is a square root of* x)) Gxy

(∀x: Nx ∧ Gxo) (∃y: (Py ∧ Syx) ∧ (∀z: ¬ z = y) ¬ (Pz ∧ Szx)) Gxy
∀x ((Nx ∧ Gxo) → ∃y ((Py ∧ Syx) ∧ ∀z (¬ z = y → ¬ (Pz ∧ Szx)) ∧ Gxy))

or:

(∀x: Nx ∧ Gxo) (∃y: (Py ∧ Syx) ∧ (∀z: Pz ∧ Szx) y = z) Gxy
∀x ((Nx ∧ Gxo) → ∃y ((Py ∧ Syx) ∧ ∀z ((Pz ∧ Szx) → y = z) ∧ Gxy))

[G: λx (x *is greater than* y); N: λx (x *is a number*); P: λx (x *is positive*); S: λxy (x *is a square root of* y)]

with the description operator:

Every number greater than one is greater than its positive square root
(∀x: x *is a number* ∧ x *is greater than one*) x *is greater than the positive square root of* x
(∀x: Nx ∧ Gxo) G x *the positive square root of* x
(∀x: Nx ∧ Gxo) Gx(℩y y *is a positive square root of* x)
(∀x: Nx ∧ Gxo) Gx(℩y (y *is a positive* ∧ y *is a square root of* x))

(∀x: Nx ∧ Gxo) Gx(℩y (Py ∧ Syx))
∀x ((Nx ∧ Gxo) → Gx(℩y (Py ∧ Syx)))

2. **a.** (∃x: x *owns Spot* ∧ (∀y: ¬ y = x) ¬ y *owns Spot*) x *called*
(∃x: x *owns Spot* ∧ *only* x *owns Spot*) x *called*
The owner of Spot is such that (it called)

Spot's owner called

b. *John found* (℩x (x *is a photographer* ∧ x *enlarged* (℩y y *is a picture of John*)))
John found (℩x (x *is a photographer* ∧ x *enlarged the picture of John*))
John found (℩x (x *is a photographer who enlarged the picture of John*))

John found the photographer who enlarged the picture of him