8.4.3. Examples: restrictive *vs.* non-restrictive relative clauses

The distinction between restrictive and non-restrictive relative clauses is a natural application of an analysis of definite descriptions. Although the significance of the distinction is not as great as it is for generalizations, it is greater for definite descriptions than it is for claims of exemplification. And, since restrictive relative clauses are part of definite descriptions but not themselves individual terms, it is only the sort of analysis of definite descriptions that we are now considering that can exhibit their role.

We will consider a single pair of sentences and analyze each of them using the two approaches to definite descriptions. Since these analyses are not equivalent, we can expect different results but, since the difference between the analyses involves a failure of normal reference, we cannot expect great differences when the descriptions work normally—i.e., when reference succeeds and true claims are made.

The two sentences we will consider are these:

> *The part that Tom requested was defective.*
> *The part, which Tom requested, was defective.*

The difference between having a restrictive relative clause in the first and a non-restrictive relative clause in the second is, intuitively, whether the relative clause contributes to the specification of what is referred to or instead to what is said about it. That difference can be emphasized by expanding the second sentence to *The part, which, by the way, Tom requested, was defective*.

We will begin with an analysis of these two sentences using the description operator. This begins as an analysis in chapter 6 would have but continues further. In the case of the first sentence, we have

> *The part that Tom requested was defective*
> *The part that Tom requested was defective*
>   D *the part that Tom requested*
>   D(Ix x *is a part that Tom requested*)
>   D(Ix (x *is a part* ∧ *Tom requested* x))
>
>   D(Ix (Px ∧ Rtx))

[D: λx (x *was defective*); P: λx (x *is a part*); R: λxy (x *requested* y); t: *Tom*]

In chapter 6, we would have ended up with something like D(pt) where p abbreviated a functor that produced the term *the part that Tom requested* when applied to the term *Tom*. Since the two expressions

> [λy (Ix (Px ∧ Ryx))]t   Ix (Px ∧ Rtx)

are really two forms of notation for the same term, we can say that the analysis

above extends the analysis of chapter 6 by analyzing the functor p as λy (ɪx (Px ∧ Ryx)). Indeed, one of the main reasons definite descriptions were of interest to Frege and Russell was their role in specifying the output of functors by way of relations since many mathematical functions are naturally defined in this way.

The analysis of the sentence with non-restrictive relative clause also begins as in chapter 6.

<div align="center">

The part, which Tom requested, was defective
Tom requested the part ∧ the part was defective
R *Tom the part* ∧ D *the part*
Rt(ɪx x *is a part*) ∧ D(ɪx x *is a part*)

Rt(ɪx Px) ∧ D(ɪx Px)

</div>

[D: λx (x *was defective*); P: λx (x *is a part*); R: λxy (x *requested* y); t: *Tom*]

To make it easier to compare the two analyses, let us reorder the conjuncts in the second to get

<div align="center">

D(ɪx Px) ∧ Rt(ɪx Px)

</div>

and then restate this using an abstract so that the definite description occurs only once

<div align="center">

[λx (Dx ∧ Rtx)](ɪx Px)

</div>

The difference between the sentences restrictive and non-restrictive clauses, when seen in this way—i.e., as

<div align="center">

D(ɪx (Px ∧ Rtx))   [λx (Dx ∧ Rtx)](ɪx Px)

</div>

—lies in the location of the predicate λx Rtx or λx (*Tom requested* x). In both cases it is used to provide a further conjunct; but, in the analysis of restrictive clause, this conjunct appears in the description to which the definite article is applied, and in the analysis of the non-restrictive clause, it appears in what is predicated of a definite description. This is the symbolic analogue of the idea that restrictive relative clause contributes to determining the reference of an individual term while a non-restrictive clause adds to what is said about the term's referent.

We can expect to find something similar when apply Russell's analysis. In the case of the first sentence, we get

The part that Tom requested was defective
The part that Tom requested is such that (it was defective)
(∃x: x *is a part that Tom requested* ∧ (∀y: ¬ y = x) ¬ y *is a part the Tom requested*) x *was defective*
(∃x: (x *is a part* ∧ *Tom requested* x) ∧ (∀y: ¬ y = x) ¬ (y *is a part* ∧ *Tom requested* y)) x was defective

(∃x: (Px ∧ Rtx) ∧ (∀y: ¬ y = x) ¬ (Py ∧ Rty)) Dx
or: (∃x: (Px ∧ Rtx) ∧ (∀y: Py ∧ Rty) x = y) Dx

[D: λx (x *was defective*); P: λx (x *is a part*); R: λxy (x *requested* y); t: *Tom*]

On Russell's analysis, the definite article *the* can be seen to mark an operation that applies to a predicate ρ to form the quantifier (∃x: ρx ∧ (∀y: ¬ y = x) ¬ ρy) or (∃x: ρx ∧ (∀y: ρy) x = y). In the sentence with the restrictive relative clause, the predicate ρ is λx (Px ∧ Rtx), and this involves the predicate λx Rtx that corresponds to the relative clause.

Russell's analysis of the sentence with a non-restrictive relative clause finds a conjunction. We must choose whether this conjunction has wider or narrower scope than the quantifier phrase associated with the definite description; but, while this is the sort of thing that leads to non-equivalent analyses in the case of negation, here the results of the two approaches are equivalent.

The part, which Tom requested, was defective
Tom requested the part ∧ the part was defective
the part is such that (Tom requested it) ∧ the part is such that (it was defective)
(∃x: x *is a part* ∧ (∀y: ¬ y = x) ¬ y *is a part*) *Tom requested* x ∧ (∃x: x *is a part* ∧ (∀y: ¬ y = x) ¬ y *is a part*) x *was defective*

(∃x: Px ∧ (∀y: ¬ y = x) ¬ Py) Rtx ∧ (∃x: Px ∧ (∀y: ¬ y = x) ¬ Py) Dx
or: (∃x: Px ∧ (∀y: Py) x = y) Rtx ∧ (∃x: Px ∧ (∀y: Py) x = y) Dx

[D: λx (x *was defective*); P: λx (x *is a part*); R: λxy (x *requested* y); t: *Tom*]

The part, which Tom requested, was defective
The part is such that (it, which Tom requested, was defective)
(∃x: x *is a part* ∧ (∀y: ¬ y = x) ¬ y *is a part*) x*, which Tom requested, was defective*
(∃x: Px ∧ (∀y: ¬ y = x) ¬ Py) (*Tom requested* x ∧ x *was defective*)

(∃x: Px ∧ (∀y: ¬ y = x) ¬ Py) (Rtx ∧ Dx)
or: (∃x: Px ∧ (∀y: Py) x = y) (Rtx ∧ Dx)

It usually makes a difference whether an existential is applied to a conjunction or to each conjunct separately because different examples may make each conjunct true and there may be no one example that would serve for both. But this will not happen with the sort of existential quantifiers used to represent definite descriptions because the restricting formula requires uniqueness. This means that if we claim the existence of an example x that satisfies this

restricting formula, part of what we have claimed is that this is the only example possible. So, when the quantifier is applied to separately in two conjuncts of a conjunctions, the conjuncts cannot be true unless there is a single example which makes both true.

Here is a table showing the simplest analyses of each of the four sorts:

	restrictive clause	non-restrictive clause
description operator	D(ɪx (Px ∧ Rtx))	Rt(ɪx Px) ∧ D(ɪx Px)
Russell's analysis	(∃x: (Px ∧ Rtx) ∧ (∀y: Py ∧ Rty) x = y) Dx	(∃x: Px ∧ (∀y: Py) x = y) (Rtx ∧ Dx)

The differences between the two sorts of relative clause are easiest to describe in the case of Russell's analysis. If we convert the analyses in the second row to unrestricted existential quantifiers and reorder conjuncts, we get the following:

restrictive clause	∃x (Px ∧ Rtx ∧ Dx ∧ (∀y: Py ∧ Rty) x = y)
non-restrictive clause	∃x (Px ∧ Rtx ∧ Dx ∧ (∀y: Py) x = y)

This reformulation makes it clear that the sentence stated using the restrictive relative clause is entailed by the sentence using the non-restrictive clause. The only difference lies in the restriction of the generalization appearing as the last conjunct of the formula to which the existential is applied. And the added restriction makes the generalization derived from the restrictive clause weaker since it says about x only that it accounts for all the parts Tom requested rather than that it accounts for all the parts whatsoever. And it is also clear for the same reason that no entailment holds in the other direction.

In the analyses using the description operator, if a description is not uniquely satisfied, the definite description has the nil reference value and whether what is said is true or false will depend on what predicates are true or false of the nil value. Each of the two sorts of clause succeeds in referring in some circumstances where the other does not: there may be more than one part but just one that Tom requested and there may be exactly one part but none that Tom requested. It follows that there will be some circumstance in which the two sentences will be talking about different things, in one case a real object and, in the other, the nil value. It is easy to find such circumstances in which the sentence with the restrictive clause is true and the one with the non-restrictive clause is false. The other direction is harder; but, if there is more than one part and Tom requested only one, which was not defective, D(ɪx (Px ∧ Rtx)) will be false and Rt(ɪx Px) ∧ D(ɪx Px) will still be true provided that Rt∗ and D∗ are true. (Notice that it must then be the case that P∗ is false if ɪx (Px ∧ Rtx) is to have a non-nil value.) Since it is also easy to find cases where the two sentences are both true or both false on when they are analyzed using the description operator, the two sentences are counted as logically independent by that analysis.

Thus, while each sort of analysis makes a distinction between the meanings of the two sentences, their accounts of this distinction are different. However, this difference does not provide much basis on which to argue for the correctness of

one analysis over the other. The difference only concerns circumstances in which a definite description is not uniquely satisfied; and, although we are assuming that a sentence containing such a description does have a truth value, there seems to be little grounds for saying what that truth value ought to be.

The most important differences between restrictive and non-restrictive clauses are probably not the differences in truth conditions that our symbolic analyses are designed to capture but instead differences in appropriateness. To be used appropriately, a definite description must do enough to identify a unique object given the context, and there seem to be also requirements governing the way this is done.

First, the description should identify an object using information that is already shared by parties to the conversation. It would be odd to use *The part that Tom requested was defective* if one's audience was not already aware that Tom had requested a part—it might prompt the response, *I didn't know Tom requested a part*—but *The part, which Tom requested, was defective* would be appropriate in this sort of case if the part in question was already sufficiently salient that it could be identified by the simple description *the part*. On the other hand, if it is known that Tom requested one and only one part but this part is not already sufficiently salient to be identified as *the part*, the sentence with the restrictive clause is appropriate but the one with the non-restrictive clause would not be. In a case like this, the added description provided by the restrictive clause might serve to distinguish one part among a group of equally salient parts or to shift attention from one part to another one.

It also seems to be a requirement for appropriateness that the various elements of a definite description actually be needed to identify an object. If a single part is salient enough to be identified by *the part* alone, then, even if everyone is aware that Tom had requested it, the sentence *The part that Tom requested was defective* will seem odd and may prompt the response *But I thought that was the part we were talking about*. The sentence with the non-restrictive clause would be no better under these circumstances but for a different reason: it would purport to introduce as new information something that was already known.

<div align="center">Glen Helman 26 Nov 2004</div>